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A new method for intuitionistic fuzzy multi-objective linear fractional optimization problem and its application in agricultural land allocation problem

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ABSTRACT

This paper presents a new method for solving an intuitionistic fuzzy multi-objective linear fractional optimization (IFMOLFO) problem with crisp and intuitionistic fuzzy constraints. Here, all uncertain parameters are represented as triangular intuitionistic fuzzy numbers. We used an accuracy ranking function and variable transformation in the proposed method to convert an IFMOLFO problem into a crisp multi-objective linear optimization problem. Then, we formulated the first phase of the weighted intuitionistic fuzzy goal programming (WIFGP) model to obtain an intuitionistic fuzzy non-dominant (IFND) solution for the IFMOLFO problem. Several strategies for obtaining an IFND solution to the IFMOLFO problem have been proposed in the literature. However, in addition to constructing the phase-I WIFGP model, this study shows that the IFND solution may not be Pareto-optimal when some of the under-deviation variables are zero. As a result, the second phase of the WIFGP model is applied to address this issue. The benefits of both models are merged to provide a novel method, unlike any other method in the literature, for producing optimal solutions that satisfy both IFND and Pareto-optimal requirements. The suggested algorithm's efficiency and reliability are demonstrated by addressing a real-life case study of an agricultural production planning problem and followed by solving a numerical example from literature.

1. Introduction

Many real-world application problems, such as inventory control, transportation, agriculture, finance, etc., involve the optimization of two or more conflicting objective functions under resource restrictions. The stated types of problems are appropriately modeled as multi-objective optimization problems. For this type of problem, the basic concept of a compromise solution can be considered instead of finding an optimal solution to satisfy all the conflicting objective functions. When all objective functions are linear fractional forms and the constraint functions are linear, this model is used to define the multi-objective linear fractional optimization (MOLFO) problem.

In the classical model of MOLFO problems, all the coefficients of objective and constraint functions and also the quantity of resources are assumed to be precisely known. However, such parameters (all or some) are not precise in reality due to

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market fluctuations, measurement errors, or unrestricted factors (traffic, climate, the demand of customers, and so forth). In this case, it is highly usual for decision-makers to be hesitant to estimate their desired level of the objective function as well as the problem parameters. In such instances, the decision-maker is forced to live with uncertainty and hesitancy. The intuitionistic fuzzy multi-objective linear fractional optimization (IFMOLFO) problem can be used to efficiently model these circumstances, [4,6,29,32,30].

In the past, numerous optimization methods have been suggested for solving single-objective linear fractional optimization (SOLFO) problems. The concept of variable transformation for finding the solution to the SOLFO problem was first suggested by Charnes and Cooper (1962) [8], and then explored further by Craven (1988) [10] and Schaible (1976) [31]. Zimmerman(1978) [35] was the first to employ fuzzy optimization for multiple objective application problems after Bellman and Zadeh (1970) [5] established the notion of fuzzy decision set theory. The fuzzy multi-objective linear fractional optimization (FMOLFO) problems were then studied by a number of researchers, [3,7,9,17,26,22,24,27,34].

Pal et al. (2003) [24] extended the fuzzy goal programming (FGP) approach suggested by Mohamed (1997) [23], to solve MOLFO problems in a fuzzy environment. Pramy (2017) [27] investigated a FMOLFO problem using graded mean ranking to reduce the problem into a crisp problem and then, based on Guzel (2013) [17] approach, the crisp problem transformed into a single objective linear optimization problem. Borza and Rambely (2021) [7] developed a suitable variable transformation to convert a FMOLFO problem into a multi-objective linear optimization (MOLO) problem and solved it with the max–min approach that was proposed by Zimmerman (1978) [35].

Most of the researchers listed above have suggested various fuzzy goal approaches for finding fuzzy non-dominant solutions. However, Lee and Li(1993) [19] presented a fuzzy non-dominant solution to the fuzzy multi-objective optimization (FMOO) problem that may not be a Pareto-optimal solution. To address these challenges, they devised a two-phase approach. Lee and Li (1993) [19] discovered a fuzzy non-dominant solution to the FMOO problem in the first phase using Zimmerman's min-operator. Then, they obtained a Pareto-optimal solution using the solution and an equal-weighted average-operator. Guu and Wu (1999) [16], modified the approach of Lee and Li (1993) [19] and suggested a two-phase approach with a different weighted average operator. Several fuzzy optimization strategies have recently been developed in the literature to find a compromise solution that satisfies both fuzzy non-dominant and Pareto-optimal requirements [12,18,28]. Jimenez and Bilbao(2009) [18] presented a general fuzzy approach based on the concept of goal programming for finding a Pareto optimal solution to the FMOO problem.

Although the fuzzy set type model is very helpful and flexible for representing a MOLFO problem with uncertainties, it does not model a problem when the imprecise aspiration-level and/or the values of parameters cause some degree of hesitation, [4,28,29]. Due to such a shortcoming, many researchers have expanded the conventional fuzzy set of core notions. Out of many generalized conventional fuzzy sets, the intuitionistic fuzzy sets (IFSs) suggested by Atanassov (1986) [4] have been industrialized to be extremely essential for dealing with uncertainty, ambiguity, and reluctance.

So far, several efforts have been made to solve different domain of MOLO problem using intuitionistic fuzzy optimization approach, [1,2,6,28,32,33]. In the first work, Anglov (1997)[2] extended the fuzzy decision environment that was suggested by Bellman and Zadeh(1970) [5] into an intuitionistic fuzzy decision environment for solving MOLO problems. Using Anglov (1997) [2] approach, Bharati and Singh (2014) [6] solved the real-life application of the agricultural land allocation problem (ALAP). Ebrahimnejed and Verdegay (2018) [13] developed a new method for solving transportation problems with intuitionistic fuzzy multiple objective functions. Singh and Yadav (2015) [32] reduced intuitionistic fuzzy multi-objective non-linear optimization problem into a crisp problem by utilizing linear ranking method and then solved using fuzzy techniques.

Very few researchers in the literature have devoted themselves to proposing solution approaches for the IFMOLFO problem where aspiration levels of objective functions and/or coefficients are intuitionistic fuzzy quantified. Rukmani and Porchelvi (2018) [29] extending the Pal et al. (2003) [24] approach to solving the IFMOLFO problem with intuitionistic fuzzy goals. El Sayed et al. (2021) [14] investigated a method for resolving a fully intuitionistic fuzzy multiple objective fractional transportation problem through Anglov (1997) [2] approach. Singh and Yadav (2016) [33] presented a method for solving a single-objective linear fractional optimization problem with triangular intuitionistic fuzzy values. An alternative methodology for resolving MOLFO with intuitionistic fuzzy parameters was introduced by Sahoo et al. (2022) [30]. To the best of the author's experienced in the problem domain, no works have been studied to date on the subject of the Pareto-optimal solution to the IFMOLFO problem with consideration of an intuitionistic fuzzy aspiration level for all objective and constraint functions, as well as all uncertain parameters being triangular intuitionistic fuzzy numbers (IFNs).

Since our proposed MOLFO problem takes all uncertain parameters in the form of triangular IFNs and the aspiration levels of all objective and some constraint functions in the intuitionistic fuzzy environments, the development of a new optimization method to solve IFMOLFO problems signifies this current work's aim and objective. Moreover, the following basic points can be used as the authors' important contributions to this research.

- In the process of modeling the problem, the membership and non-membership functions to measure the level of acceptance and rejection of each objective and constraint function in terms of achieving the desired levels of intuitionistic fuzzy goals are used.
- In order to find an intuitionistic fuzzy non-dominant (IFND) solution to the proposed problem, a Phase-I weighted intuitionistic fuzzy goal programming (WIFGP) model was developed in this study.

- This study also justified that the IFND solution generated from the phase-I WIFGP model is not guaranteed to be a Pareto-optimal solution to IFMOLFO problems if this optimal solution is not unique or some of the under-deviance variables are zero.
- To overcome the difficulty of obtaining a Pareto-optimal solution, a Phase-II WIFGP model is presented in this work and its results are compared with existing methods that have been suggested by researchers for solving the IFMOLFO problem. Thus, the proposed optimal solution is more effective and satisfies the decision-makers.
- The strength and flexibility of this proposed approach are enhanced by the combination of intuitionistic fuzzy optimization concepts, weighted interactive goal programming techniques, and the two-phase WIFGP model.

The remaining sections of the present work are arranged as follows: The basic definition of some pre-request concepts for this study is presented in Section 2. In Section 3 the formulation of the IFMOLFO problem, the crisp formulation of IFMOLFO problem, as well as the linear model of the IFMOLFO problem are illustrated. The explanation of the proposed solution approach is demonstrated in Section 4 and its solution approach algorithm is explained in Section 5. In Section 6 the efficiency and practicality of the proposed method are illustrated by a suitable real-case study of agricultural land allocation problem (ALAP) in Hawassa-Zuria, Sidama Region, Ethiopia and by a well known numerical example. In the final section, conclusions and future study areas are presented.

2. Preliminaries and basic concepts

Some preliminary findings and fundamental concepts needed for this study are briefly discussed in this section, which is based on the literature, [4,14,28–30,33].

Definition 1. Let X be a general set of discourse x. The collection of an ordered triplet point $\widetilde{A}^I = \left\{\left(x, \mu_{\widetilde{A}^I}(x), \nu_{\widetilde{A}^I}(x)\right) : x \in X\right\}$ is an *intuitionistic fuzzy set (IFS)*, where, the functions $\mu_{\widetilde{A}^I}(x) : X \to [0,1]$and $\nu_{\widetilde{A}^I}(x) : X \to [0,1]$are used to denote values for the degree of membership and non-membership of $x \in \widetilde{A}^I$, respectively, with the properties $0 \leqslant \mu_{\widetilde{A}^I}(x) + \nu_{\widetilde{A}^I}(x) \leqslant 1$ for all x in X. The value for the degree of hesitancy which $x \in \widetilde{A}^I$ or $x \notin \widetilde{A}^I$ is denoted by $\pi_{\widetilde{A}^I}(x)$ and defined as: $\pi_{\widetilde{A}^I}(x) = 1 - \mu_{\widetilde{A}^I}(x) - \nu_{\widetilde{A}^I}(x)$.

Definition 2. An *intuitionistic fuzzy number (IFN)* $\tilde{c}^I$is a special class of intuitionistic fuzzy set on $\Re$($\Re$ represents the set of real numbers), whose membership function $\mu_{\tilde{c}^I} : \Re \to [0,1]$and non-membership function $\nu_{\tilde{c}^I} : \Re \to [0,1]$ satisfy the basic four conditions $(1) - (4)$ that stated as follows:

(1) Normal, $\exists x_0 \in \Re$such that $\mu_{\tilde{c}^I}(x_0) = 1$ and $\nu_{\tilde{c}^I}(x_0) = 0$.
(2) The membership function $\mu_{\tilde{c}^I}$is quasi-concave; that means, for any $x_1, x_2 \in \Re, \mu_{\tilde{c}^I}(\lambda x_1 + (1-\lambda)x_2) \geqslant min\{\mu_{\tilde{c}^I}(x_1), \mu_{\tilde{c}^I}(x_2)\} \forall \lambda \in [0,1]$ and $\mu_{\tilde{c}^I} : \Re \to [0,1]$is piecewise continuous on $\Re$.
(3) The non-membership function $\nu_{\tilde{c}^I}$is quasi-convex; that means, $x_1, x_2 \in \Re, \nu_{\tilde{c}^I}(\lambda x_1 + (1-\lambda)x_2) \leqslant max\{\nu_{\tilde{c}^I}(x_1), \nu_{\tilde{c}^I}(x_2)\} \forall \lambda \in [0,1]$ and $\nu_{\tilde{c}^I} : \Re \to [0,1]$piecewise continuous on $\Re$.
(4) The support of $\tilde{c}^I$ (i.e.,$\tilde{c}^I_{<0,1>} = \{x \in R | \mu_{\tilde{c}^I}(x) \geqslant 0, \nu_{\tilde{c}^I}(x) \leqslant 1\}$) is compact.

Definition 3. *Triangular Intuitionistic Fuzzy Number (triangular IFN)* is a special class of IFS on $\Re$ and is denoted as $\tilde{a}^I = (\underline{a}, a, \bar{a}; \underline{b}, a, \bar{b})$ where, $\underline{a}, a, \bar{a}, \underline{b}, \bar{b} \in \Re$ such that $\underline{b} \leqslant \underline{a} \leqslant a \leqslant \bar{a} \leqslant \bar{b}$. This number may represent the value of an imprecise data like "virtually a", which is "almost equal to a". These means, the most adequate value is "*a*".

The membership function, $\mu_{\tilde{a}^I}(x)$ and non-membership function, $\nu_{\tilde{a}^I}(x)$ of triangular IFN $\tilde{a}^I = (\underline{a}, a, \bar{a}; \underline{b}, a, \bar{b})$ can be illustrate in Fig. 1 and defined as follows:

$$\mu_{\tilde{a}^I}(x) = \begin{cases} \frac{x-\underline{a}}{a-\underline{a}} & \text{if } \underline{a} \leqslant x < a \\ 1 & \text{if } x = a \\ \frac{\bar{a}-x}{\bar{a}-a} & \text{if } a < x \leqslant \bar{a} \\ 0 & \text{if } x < \underline{a}, x > \bar{a} \end{cases} \quad \nu_{\tilde{a}^I}(x) = \begin{cases} \frac{a-x}{a-\underline{b}} & \text{if } \underline{b} \leqslant x < a \\ 0 & \text{if } x = a \\ \frac{x-a}{\bar{b}-a} & \text{if } a < x \leqslant \bar{b} \\ 1 & \text{if } x < \underline{b}, x > \bar{b} \end{cases} \tag{1}$$

Definition 4. *Arithmetic Operation over triangular IFN:* Given two arbitrary triangular IFN $\tilde{a}^I = (\underline{a}, a, \bar{a}; \underline{b}, a, \bar{b})$ and $\tilde{c}^I = (\underline{c}, c, \bar{c}; \underline{d}, c, \bar{d}), \lambda \in \Re$.

1. Addition: $\tilde{a}^I + \tilde{c}^I = (\underline{a} + \underline{c}, a + c, \bar{a} + \bar{c}; \underline{b} + \underline{d}, a + c, \bar{b} + \bar{d})$

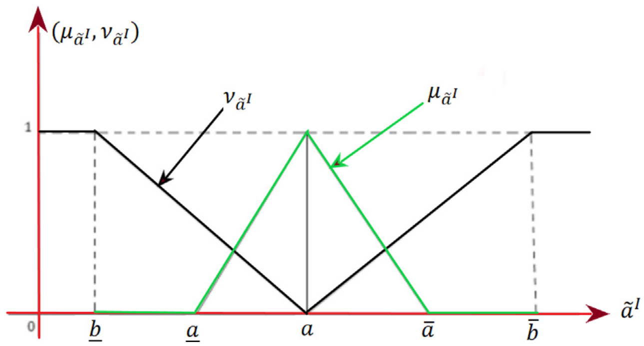

Fig. 1. Triangular Intuitionistic Fuzzy Number, $\tilde{a}^I = \left(\underline{a}, a, \bar{a}; \underline{b}, a, \bar{b}\right)$.

2. Subtraction: $\tilde{a}^I - \tilde{c}^I = \left(\underline{a} - \bar{c}, a - c, \bar{a} - \underline{c}; \underline{b} - \bar{d}, a - c, \bar{b} - \underline{d}\right)$
3. Scalar Multiplication: $\lambda \times \tilde{a}^I = \begin{cases} (\lambda\underline{a}, \lambda a, \lambda\bar{a}; \lambda\underline{b}, \lambda a, \lambda\bar{b}) & \text{if } \lambda > 0 \\ (\lambda\bar{a}, \lambda a, \lambda\underline{a}; \lambda\bar{b}, \lambda a, \lambda\underline{b}) & \text{if } \lambda < 0 \\ (0,0,0;0,0,0) = 0 & \text{if } \lambda = 0 \end{cases}$
4. Multiplication: $\tilde{a}^I \times \tilde{c}^I = \begin{cases} (\underline{a}\,\underline{c}, ac, \bar{a}\bar{c}; \underline{b}\,\underline{d}, ac, \bar{b}\bar{d}) & \text{if } \tilde{a} > 0, \tilde{c} > 0 \\ (\underline{a}\bar{c}, ac, \bar{a}\underline{c}; \bar{b}\underline{d}, ac, \bar{b}\underline{d}) & \text{if } \tilde{a} < 0, \tilde{c} > 0 \\ (\bar{a}\bar{c}, ac, \underline{a}\,\underline{c}; \bar{b}\bar{d}, ac, \underline{b}\,\underline{d}) & \text{if } \tilde{a} < 0, \tilde{c} < 0 \end{cases}$

Definition 5. Let a triangular IFN($\Re$) be given as $\tilde{a}^I = \left(\underline{a}, a, \bar{a}; \underline{b}, a, \bar{b}\right)$. Then the score function of $\tilde{a}^I$ for the membership and non membership function, denoted by $E^{\mu}(\tilde{a}^I)$ and $S^{\nu}(\tilde{a}^I)$, respectively, are defined as:

$$E^{\mu}(\tilde{a}^I) = \frac{\underline{a}+\bar{a}+2a}{4} \text{ and } S^{\nu}(\tilde{a}^I) = \frac{\underline{b}+\bar{b}+2a}{4} \quad (2)$$

The accuracy function of triangular IFN $\tilde{a}^I$ is the weighted mean of score functions of $\tilde{a}^I$, which is denoted by $A^{\lambda}(\tilde{a}^I)$, and developed by the expression: $A^{\lambda}(\tilde{a}^I) = \lambda E^{\mu}(\tilde{a}^I) + (1-\lambda)S^{\nu}(\tilde{a}^I)$with a weight $\lambda \in [0,1]$ set by decision-maker (DM).

Proposition 2.1. The accuracy function $A^{\lambda} : IFN(\Re) \rightarrow \Re$ is a linear function over any two triangular IFN $\tilde{a}^I, \tilde{b}^I$. That means; for any $\tau \in \Re$, we have $A^{\lambda}\left(\tau\tilde{a}^I + \tilde{b}^I\right) = \tau A^{\lambda}(\tilde{a}^I) + A^{\lambda}\left(\tilde{b}^I\right)$.

Proposition 2.2. 1. Let s be any crisp number, then $A^{\lambda}(s) = s$.

2. For any real constant number $0 < r < s$, the triangular IFN $\tilde{a}^I = (a - r, a, a + r; a - s, a, a + s)$ is called Symmetric triangular IFN and $A^{\lambda}(\tilde{a}^I) = a$.

The proof of Propositions 2.1 and 2.2 are available in the literature, [4,14,28,33].

Definition 6. *Ranking of Accuracy Functions:* Assume that for any two arbitrary triangular IFN($\Re$), $\tilde{a}^I$ and $\tilde{c}^I$ have accuracy functions $A^{\lambda}(\tilde{a}^I)$ and $A^{\lambda}(\tilde{c}^I)$ for any weight $\lambda \in [0,1]$. Then the ranking order of triangular IFN($\Re$), $\tilde{a}^I$ and $\tilde{c}^I$ are specified in the following ways:

i. If the accuracy functions of $\tilde{a}^I$ and $\tilde{c}^I$ is $A^{\lambda}(\tilde{a}^I) > A^{\lambda}(\tilde{c}^I)$, then $\tilde{a}^I > \tilde{c}^I$.
ii. If the accuracy functions of $\tilde{a}^I$ and $\tilde{c}^I$ is $A^{\lambda}(\tilde{a}^I) < A^{\lambda}(\tilde{c}^I)$,then $\tilde{a}^I < \tilde{c}^I$.
iii. If the accuracy functions of $\tilde{a}^I$ and $\tilde{c}^I$ is $A^{\lambda}(\tilde{a}^I) = A^{\lambda}(\tilde{c}^I)$, then $\tilde{a}^I = \tilde{c}^I$.

3. Formulation of intuitionistic fuzzy multi-objective linear fractional optimization problem

The mathematical form of an intuitionistic fuzzy multi-objective linear fractional optimization (IFMOLFO) problem can be summarized as follows:

$$\tilde{max}\ \tilde{z}^I(\bar{x}) = \left(\frac{\widetilde{N}^I_1(\bar{x})}{\widetilde{D}^I_1(\bar{x})}, \frac{\widetilde{N}^I_2(\bar{x})}{\widetilde{D}^I_2(\bar{x})}, \ldots, \frac{\widetilde{N}^I_T(\bar{x})}{\widetilde{D}^I_T(\bar{x})}\right)$$
$$\text{Subject to :}\qquad \bar{x} \in \widetilde{S}^I = \left\{\begin{array}{ll} \tilde{g}^I_i(\bar{x}) = \tilde{a}^I_i\bar{x}\ (\tilde{\leqslant}, \tilde{\geqslant}, or\ \tilde{=})\ \tilde{b}^I_i, & i = 1,2,3,\ldots,p; \\ g_j(\bar{x}) = a_j\bar{x}\ (\geqslant, \leqslant\ or\ =)b_j, & j = p+1, p+2,\ldots,m; \\ \bar{x} = (x_1, x_2, \ldots, x_n) \geqslant \bar{0} & \end{array}\right\} \tag{3}$$

Where,

(i)$\tilde{z}^I_t(\bar{x}) = \frac{\widetilde{N}^I_t(\bar{x})}{\widetilde{D}^I_t(\bar{x})}$ is an intuitionistic fuzzy linear fractional objective function, $\tilde{g}^I_i(\bar{x})$ and $g_j(\bar{x})$ are intuitionistic fuzzy and crisp linear constraint functions, respectively.
(ii) $\widetilde{N}^I_t(\bar{x}) = \tilde{c}^I_t\bar{x} + \tilde{\alpha}^I_t, \widetilde{D}^I_t(\bar{x}) = \tilde{d}^I_t\bar{x} + \tilde{\beta}^I_t > 0\ \forall\bar{x} \in S$, foreach $t = 1,2,3,\ldots,T$.
(iii)The decision vectors are $\bar{x} = (x_1, x_2, \ldots, x_n) \in \Re^n$.
(iv) $\widetilde{S}^I$ is a non-empty intuitionistic fuzzy bounded and convex constrain set.
(v) The value of parameters $\tilde{c}^I_t, \tilde{d}^I_t, \tilde{\alpha}^I_t, \tilde{\beta}^I_t, \tilde{a}^I_i$ and $\tilde{b}^I_i$ are expressed as triangular IFNs.
(vii) The IFMOLFO (3) is concave-convex programming problem i.e. $\widetilde{N}^I_t(\bar{x})$ is concave function with $\widetilde{N}^I_t(\bar{x}) \geqslant 0$ for some $\bar{x} \in \widetilde{S}^I$ and $\widetilde{D}^I_t(\bar{x})$ is convex function with $\widetilde{D}^I_t(\bar{x}) > 0, \forall\bar{x} \in \widetilde{S}^I$.

Definition 7. An intuitionistic fuzzy non-dominant solution, $\bar{x}^* = (x^*_1, x^*_2, \ldots, x^*_n)$ is Pareto optimal to IFMOLFO (3), if there does not exist ($\nexists$) another $\bar{x} \in S^I$ such that $\tilde{z}^I_t(\bar{x}) \geqslant \tilde{z}^I_t(\bar{x}^*), \forall t$ and strictly inequality hold for some t.

3.1. Crisp multi-objective linear fractional optimization problem formulation

Using the accuracy ranking method, we reduce every triangular intuitionistic fuzzy parameters in the IFMOLFO (3) to crisp parameters. Thus, the crisp MOLFO (4) equivalent to IFMOLFO (3) is formulated as follows:

$$max\ z(\bar{x}) = \left(\frac{N_1(\bar{x})}{D_1(\bar{x})}, \frac{N_2(\bar{x})}{D_2(\bar{x})}, \ldots, \frac{N_T(\bar{x})}{D_T(\bar{x})},\right.$$
$$\text{Subject to :}\qquad \bar{x} \in S^c = \left\{\begin{array}{l} g_i(\bar{x}) = a_i\bar{x}\ (\tilde{\leqslant}, \tilde{\geqslant}, or\ \tilde{=})\ b_i, i = 1,2,3,\ldots,p; \\ g_j(\bar{x}) = a_j\bar{x}\ (\geqslant, \leqslant\ or\ =)b_j, j = p+1, p+2,\ldots,m; \\ \bar{x} = (x_1, x_2, \ldots, x_n) \geqslant \bar{0} \end{array}\right\} \tag{4}$$

Where $N_t = A^\lambda\left(\widetilde{N}^I_t\right), D_t = A^\lambda\left(\widetilde{D}^I_t\right), z = A^\lambda(\tilde{z}^I), t = 1,2,3,\ldots T; g_i(\bar{x}) = A^\lambda(\tilde{g}^I_i(\bar{x})), b_i = A^\lambda\left(\tilde{b}^I_i\right) i = 1,2,3,\ldots,p$ with accuracy function operator, A^λ. Still, the relation between $g_i(\bar{x})$ and b_i are treated under intuitionistic fuzzy environment.

Theorem 3.1. *A Pareto-optimal solution for the crisp MOLFO (4) is a Pareto-optimal solution for the IFMOLFO (3).*

Proof 1. To prove this theorem first, we need to show that every feasible solution $\bar{x} \in S^c$ of crisp MOLFO (4) is also a feasible solution of the IFMOLFO (3), i.e prove that $\bar{x} \in S^I$. Let $\bar{x} \in S^c = \{\bar{x} \in \Re^n : g_i(\bar{x})\ (\tilde{\leqslant}, \tilde{\geqslant}, or\ \tilde{=})\ b_i,\ i = 1,2,3,\ldots,p;\ g_j(\bar{x})(\geqslant, \leqslant\ or\ =)b_j, j = p+1, p+\ 2,\ldots,m; \bar{x} \geqslant \bar{0}\}$. Since $g_i = A^\lambda(g^I_i)$ and $b_i = A^\lambda\left(b^I_i\right)$, we get, $S^c = \left\{\bar{x} \in \Re^n : A^\lambda(g^I_i)(\bar{x})\ (\tilde{\leqslant}, \tilde{\geqslant}, or\ \tilde{=})\ A^\lambda\left(b^I_i\right); g_j(\bar{x})(\geqslant, \leqslant\ or\ =)b_j; \bar{x} \geqslant \bar{0}\right\}$. Since A^λ is a linear function (proposition:-2.1) and other constraints in $\widetilde{S}^I$ are crisp (see proposition:-2.2), we have $\left\{\bar{x} \in \Re^n : A^\lambda\left(g^I_i(\bar{x})\ (\tilde{\leqslant}, \tilde{\geqslant}, or\ \tilde{=})\ b^I_i\right); g_j(\bar{x})(\geqslant, \leqslant\ or\ =)b_j; \bar{x} \geqslant \bar{0}\right\} \Rightarrow \left\{\bar{x} \in \Re^n : \tilde{g}^I_i(\bar{x})\ (\tilde{\leqslant}, \tilde{\geqslant}, or\ \tilde{=})\ \tilde{b}^I_i; g_j(\bar{x})(\geqslant, \leqslant\ or\ =)b_j; \bar{x} \geqslant \bar{0}\right\}, \Rightarrow \bar{x} \in S^I\ \left(\Rightarrow S^c \subset \widetilde{S}^I\right)$.Therefore; $\bar{x} = (x_1, x_2, \ldots, x_n)$ is a feasible solution to IFMOLFO (3). Next, we need to show that $\bar{x}^*$ is a Pareto-optimal solution to IFMOLFO (3). Since $\bar{x}^*$ is Pareto-optimal solution of crisp MOLFO (4), then $\nexists\bar{x} \neq \bar{x}^* \in S^c$ such that $z_t(\bar{x}) \geqslant z_t(\bar{x}^*), \forall t = 1,2,3,\ldots,T$ and $z_t(\bar{x}) > z_t(\bar{x}^*)$, for at least one t. Thus we have $\nexists\bar{x} \neq \bar{x}^* \in S^c$ such that $\frac{N_t(\bar{x})}{D_t(\bar{x})} \geqslant \frac{N_t(\bar{x}^*)}{D_t(\bar{x}^*)}, \forall t = 1,2,3,\ldots,T$ and $\frac{N_t(\bar{x})}{D_t(\bar{x})} > \frac{N_t(\bar{x}^*)}{D_t(\bar{x}^*)}$ for at least one t. Let $N_t = A^\lambda\left(\widetilde{N}^I_t\right)$ and $D_t = A^\lambda\left(\widetilde{D}^I_t\right)$, we get $\frac{A^\lambda\left(\widetilde{N}^I_t\right)(\bar{x})}{A^\lambda\left(\widetilde{D}^I_t\right)(\bar{x})} \geqslant \frac{A^\lambda\left(\widetilde{N}^I_t\right)(\bar{x}^*)}{A^\lambda\left(\widetilde{D}^I_t\right)(\bar{x}^*)}, \forall t = 1,2,3,\ldots,T$ and $\frac{A^\lambda\left(\widetilde{N}^I_t\right)(\bar{x})}{A^\lambda\left(\widetilde{D}^I_t\right)(\bar{x})} > \frac{A^\lambda\left(\widetilde{N}^I_t\right)(\bar{x}^*)}{A^\lambda\left(\widetilde{D}^I_t\right)(\bar{x}^*)}$for at least one t. Since A^λ is linear function (proposition:-2.1), and $S^c \subset \widetilde{S}^I$, so we obtain $\frac{\widetilde{N}^I_t(\bar{x})}{\widetilde{D}^I_t(\bar{x})} \geqslant \frac{\widetilde{N}^I_t(\bar{x}^*)}{\widetilde{D}^I_t(\bar{x}^*)}, \forall t = 1,2,3,\ldots,T$ and $\frac{\widetilde{N}^I_t(\bar{x})}{\widetilde{D}^I_t(\bar{x})} > \frac{\widetilde{N}^I_t(\bar{x}^*)}{\widetilde{D}^I_t(\bar{x}^*)}$ for at least one t. Therefore, $\nexists\bar{x} \neq \bar{x}^* \in \widetilde{S}^I$ such that $\tilde{z}^I_t(\bar{x}) \geqslant \tilde{z}^I_t(\bar{x}^*), \forall t = 1,2,3,\ldots,T$ and $\tilde{z}^I_t(\bar{x}) > \tilde{z}^I_t(\bar{x}^*)$, for at least one t. Hence $\bar{x}^*$ is Pareto-optimal solution for IFMOLFO (3). □

3.2. Linear model formulation of the MOLFO problem

In this subsection, strategies for the formulation of the equivalent crisp multi-objective linear optimization (MOLO) (6) model of the MOLFO (4) are addressed, based on the linear transformation presented by Charnes and Cooper (1962) [8] and then extended by [26,33,34]. Let r be the smallest value of $\frac{1}{D_t(\bar{x})} > 0$ for $t = 1, 2, 3, \ldots, T$. That is, $r = \cap_{t=1}^{T} \frac{1}{D_t(\bar{x})}$ which is the same to

$$rD_t(\bar{x}) \leqslant 1;\ t = 1, 2, 3, \ldots, T. \tag{5}$$

By taking the variable transformation $\bar{y} = r\bar{x}, r > 0$ and Eqs. (5), the MOLFO (4) is transformed into an equivalent to MOLO (6) model as follows:

$$\max Z(\bar{y}, r) = \left\{rN_1\left(\tfrac{\bar{y}}{r}\right), rN_2\left(\tfrac{\bar{y}}{r}\right), \ldots, rN_T\left(\tfrac{\bar{y}}{r}\right)\right\}$$
$$\text{Subject to :}\qquad (\bar{y}, r) \in S = \left\{\begin{array}{l} g_i\left(\frac{\bar{y}}{r}\right) - b_i\left(\tilde{\leqslant}, \tilde{\geqslant}\ or\ \tilde{=}\right)0;\ i = 1, 2, 3, \ldots, p; \\ g_j\left(\frac{\bar{y}}{r}\right) - b_j\left(\geqslant, \leqslant\ or\ =\right)0, j = p+1, p+2, \ldots, m; \\ rD_1\left(\frac{\bar{y}}{r}\right) \leqslant 1; rD_2\left(\frac{\bar{y}}{r}\right) \leqslant 1, \ldots, rD_T\left(\frac{\bar{y}}{r}\right) \leqslant 1 \\ r > 0, \bar{y} \geqslant 0. \end{array}\right\} \tag{6}$$

Theorem 3.2. *If the solution $(\bar{y}^*, r^*)$ is Pareto-optimal solution for the model of MOLO (6) with $N(\eta) \geqslant 0$ for some $\eta \in S$, then the corresponding $\bar{x}^*$ (by back ward substitution $\bar{x}^* = \frac{\bar{y}^*}{r^*}$) is also a Pareto-optimal solution for the model of IFMOLFO (3).*

Proof 2. Suppose that $(\bar{y}^*, r^*)$ is Pareto-optimal solution to MOLO (6) model, so we have $\forall(\bar{y}, r) \in S, rN_t\left(\frac{\bar{y}}{r}\right) \leqslant r^* N_t\left(\frac{\bar{y}^*}{r^*}\right), \forall t$ and also $rN_t\left(\frac{\bar{y}}{r}\right) < r^* N_t\left(\frac{\bar{y}^*}{r^*}\right)$ for some t. Assume that $\bar{x}^*$ is not Pareto-optimal solution to MOLFO (4), then there exist another feasible solution $\bar{x}$ such that $z_t(\bar{x}^*) \leqslant z_t(\bar{x}), \forall t$ and $z_t(\bar{x}^*) < z_t(\bar{x})$ for some t. This implies, $\frac{N_t(\bar{x}^*)}{D_t(\bar{x}^*)} \leqslant \frac{N_t(\bar{x})}{D_t(\bar{x})}, \forall t$ and $\frac{N_t(\bar{x}^*)}{D_t(\bar{x}^*)} < \frac{N_t(\bar{x})}{D_t(\bar{x})}$ for some t. Using the variable transformation $\bar{x}^* = \frac{\bar{y}^*}{r^*}, r^* = \frac{1}{D_t(\bar{x}^*)}, \bar{x} = \frac{\bar{y}}{r}, \& r = \frac{1}{D_t(\bar{x})}$suggested by Charnes and Cooper (1962) [8], we presume that: $r^* N_t\left(\frac{\bar{y}^*}{r^*}\right) \leqslant rN_t\left(\frac{\bar{y}}{r}\right), \forall t$ and also, $r^* N_t\left(\frac{\bar{y}^*}{r^*}\right) < rN_t\left(\frac{\bar{y}}{r}\right)$ for some t. Hence, we arrived at a contradiction with $(\bar{y}^*, r^*)$ as Pareto-optimal solution to MOLO (6) model. Therefore using the above theorem-3.1, $\bar{x}^*$ is also a Pareto-optimal solution for the model of IFMOLFO (3). □

4. Proposed solution approach

To solve the developed crisp MOLO (6) model with real decision-making situations, intuitionistic fuzzy environments have the most significant role and favorable characteristics in making decisions. The intuitionistic fuzzy goal programming model, which is equivalent to the crisp MOLO (6) model in the intuitionistic fuzzy decision environment, is formulated as follows:

$$\text{Find}\quad \bar{y}, r\ \text{Subject to :} \left\{\begin{array}{ll} z_t(\bar{y}, r) \tilde{\geqslant} Z_t^A; t = 1, 2, 3, \ldots, T; & \bar{y} \in S \\ g_i(\bar{y}, r) \tilde{\geqslant} b_i^A;\ i = 1, 2, 3, \ldots, k; & \\ g_i(\bar{y}, r) \tilde{\leqslant} b_i^A;\ i = k+1, k+2, \ldots, p; & r > 0 \end{array}\right\} \tag{7}$$

Where Z_t^A and b_i^A are the aspiration levels of each intuitionistic fuzzy goal set by decision-makers for objective $z_t(\bar{y}, r)$ and constraint $g_i(\bar{y}, r)$ functions, respectively. $\tilde{\geqslant}, \tilde{\leqslant}$ used to represent an intuitionistic fuzzy inequalities. Such types of intuitionistic fuzzy goals can be treated using membership and non-membership functions.

4.1. Membership and non-membership functions

Let the membership $\mu_t(z_t(\bar{y}, r))$, $\mu_i(g_i(\bar{y}, r))$ and non-membership $v_t(z_t(\bar{y}, r)), v_i(g_i(\bar{y}, r))$ functions used to measure the degree of satisfaction/acceptance and dissatisfaction/rejection for intuitionistic fuzzy goals of objective $z_t(\bar{y}, r), t = 1, 2, 3, \ldots, T$ and constraint $g_i(\bar{y}, r), i = 1, 2, 3, \ldots, p$ functions. To construct these functions, tolerance and goals should be given first. But, it could be difficult to define them without meaningful and enough information. Therefore, to set the goals and tolerance, we need to calculate the upper and lower bounds. To find the upper and lower bounds, we solve the crisp MOLO (6) model by considering a single objective function at a time and ignoring all others until all objective functions are solved.

Let $Z_t^M(\bar{y}_t^*, r_t^*) = \max\limits_{(\bar{y}, r) \in S} z_t(\bar{y}, r)$ and $Z_t^m(\bar{y}_t^*, r_t^*) = \min\limits_{(\bar{y}, r) \in S} z_t(\bar{y}, r)$ be respectively the maximum and minimum values of each objective function under the given constraints S and $(\bar{y}_t^*, r^*)$, $t = 1, 2, 3, \ldots, T$ be an ideal solution to t^{th}-objective function. Now, using the concept of payoff matrix approach, the upper bound U_t and the lower bound L_t are the maximum and minimum values from each columns, respectively, Bharati and Singh(2014) [6], Rukmani and Porchelvi (2018) [29]. That means: $U_t = max\{z_t(\bar{y}_1^*, r_1^*), z_t(\bar{y}_2^*, r_2^*), \ldots, z_t(\bar{y}_T^*, r_T^*)\}$ and $L_t = min\{z_t(\bar{y}_1^*, r_1^*), z_t(\bar{y}_2^*, r_2^*), \ldots, z_t(\bar{y}_T^*, r_T^*)\}$, for each $t = 1, 2, 3, \ldots, T$. Therefore, the upper and lower bound for t^{th}-objective function under intuitionistic fuzzy environment can be calculated as follows:

$$\begin{aligned} &U_t^\mu = U_t; \quad L_t^\mu = L_t \rightarrow \quad \text{For membership function} \\ &U_t^\nu = U_t - \epsilon_t(U_t - L_t); \quad L_t^\nu = L_t, \epsilon_t \in (0,1) \rightarrow \text{For non-membership function} \\ &\text{The numeric value of } \epsilon_t \in (0,1) \, \textit{is first defined by decision-makers (DMs)}. \end{aligned} \tag{8}$$

Remark 4.1. The decision-makers may use the values of goals instead of U_t^μ.

Now, for the maximization type of problem, the level of satisfaction and dissatisfaction of the decision-makers will increase and decrease, respectively, as the value of objective functions tends towards U_t as shown in Fig. 2. In short: $\lim_{z_t(\bar{y},r)\rightarrow U_t^\mu} \mu_t(z_t(\bar{y},r)) = 1$, $\lim_{z_t(\bar{y},r)\rightarrow U_t^\nu} \nu_t(z_t(\bar{y},r)) = 0$. Based on this concept, the membership, $\mu_t(z_t(\bar{y},r)), t = 1, 2, 3, \ldots, T$ and non-membership, $\nu_t(z_t(\bar{y},r)), t = 1, 2, 3, \ldots, T$ functions of objective functions, $z_t(\bar{y},r)$ can be illustrated in the Fig. 2 and defined as follows for maximization type of problem:

$$\mu_t(z_t(\bar{y},r)) = \begin{cases} 0 & \text{if } z_t(\bar{y},r) \leqslant L_t^\mu \\ \frac{z_t(\bar{y},r) - L_t^\mu}{U_t^\mu - L_t^\mu} & \text{if } L_t^\mu \leqslant z_t(\bar{y},r) \leqslant U_t^\mu \\ 1 & \text{if } z_t(\bar{y},r) \geqslant U_t^\mu \end{cases} \nu_t(z_t(\bar{y},r)) = \begin{cases} 1 & \text{if } z_t(\bar{y},r) \leqslant L_t^\nu \\ \frac{U_t^\nu - z_t(\bar{y},r)}{U_t^\nu - L_t^\nu} & \text{if } L_t^\nu \leqslant z_t(\bar{y},r) \leqslant U_t^\nu \\ 0 & \text{if } z_t(\bar{y},r) \geqslant U_t^\nu \end{cases} \tag{9}$$

By setting the acceptance, q_i^{acc} and rejection, q_i^{rej} tolerance limit with $0 \leqslant q_i^{rej} < q_i^{acc}$ for $g_i(\bar{y},r) \tilde{\geqslant} b_i^A$, $i = 1, 2, 3, \ldots, k$ in the model (7), its membership, $\mu_i(g_i(\bar{y},r))$ and non-membership, $\nu_i(g_i(\bar{y},r))$ functions are defined as:

$$\mu_i(g_i(\bar{y},r)) = \begin{cases} 1 & \text{if } g_i(\bar{y},r) \geqslant b_i \\ \frac{g_i(\bar{y},r) - \left(b_i - q_i^{acc}\right)}{q_i^{acc}} & \text{if } b_i - q_i^{acc} \leqslant g_i(\bar{y},r) \leqslant b_i \\ 0 & \text{if } g_i(\bar{y},r) \leqslant b_i - q_i^{acc} \end{cases} \tag{10}$$

$$\nu_i(g_i(\bar{y},r)) = \begin{cases} 0 & \text{if } g_i(\bar{y},r) \geqslant b_i - q_i^{rej} \\ \frac{b_i - q_i^{rej} - g_i(\bar{y},r)}{q_i^{acc} - q_i^{rej}} & \text{if } b_i - q_i^{acc} \leqslant g_i(\bar{y},r) \leqslant b_i - q_i^{rej} \\ 1 & \text{if } g_i(\bar{y},r) \leqslant b_i - q_i^{acc} \end{cases} \tag{11}$$

Similar, for the case of an intuitionistic fuzzy inequality $g_i(\bar{y},r) \tilde{\leqslant} b_i^A$, $i = k+1, k+2, \ldots, p$ in the model (7), its $\mu_i(g_i(\bar{y},r))$ and $\nu_i(g_i(\bar{y},r))$ are defined as:

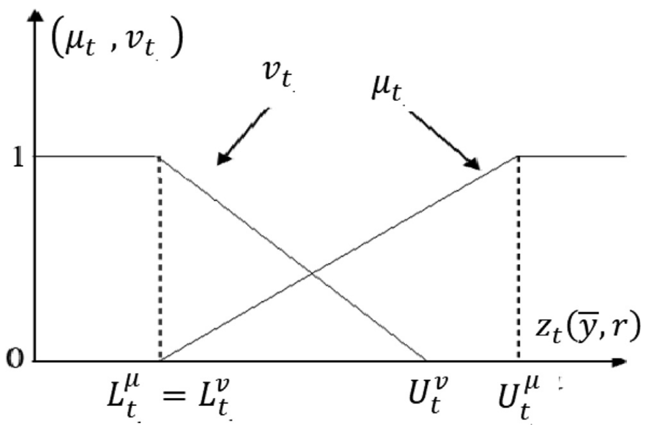

Fig. 2. Membership and non-membership function of the objective function, $z_t(\bar{y},r)$.

$$\mu_i(g_i(\bar{y},r)) = \begin{cases} 1 & \text{if } g_i(\bar{y},r) \leqslant b_i \\ \frac{b_i+q_i^{acc}-g_i(\bar{y},r)}{q_i^{acc}} & \text{if } b_i \leqslant g_i(\bar{y},r) \leqslant b_i+q_i^{acc} \\ 0 & \text{if } g_i(\bar{y},r) \geqslant b_i+q_i^{acc} \end{cases} \tag{12}$$

$$\nu_i(g_i(\bar{y},r)) = \begin{cases} 0 & \text{if } g_i(\bar{y},r) \geqslant b_i+q_i^{rej} \\ \frac{g_i(\bar{y},r)-\left(b_i+q_i^{rej}\right)}{q_i^{acc}-q_i^{rej}} & \text{if } b_i+q_i^{rej} \leqslant g_i(\bar{y},r) \leqslant b_i+q_i^{acc} \\ 1 & \text{if } g_i(\bar{y},r) \geqslant b_i+q_i^{acc} \end{cases} \tag{13}$$

4.2. Two-phase weighted intuitionistic fuzzy goal programming model

Each decision-maker's (DM's) goal in an intuitionistic fuzzy environment is to achieve the highest level of aspiration. As a result, the standard goal programming approach is employed to reduce the undesired deviation variables that prevent intuitionistic fuzzy goals from reaching their aspiration levels. The proposed method measures intuitionistic fuzzy goals (IFGs) by the linear membership and non-membership functions for t^{th} objective functions, Eqs. (9), and constraint functions, $g_i(\bar{y},r)$, as shown in Eqs. (10-13), and converts them into flexible goals with one and zero respectively, by using over-deviation $D_t^{+\mu}, d_i^{+\mu}, D_t^{+\nu}, d_i^{+\nu}$ and under-deviation $D_t^{-\mu}, d_i^{-\mu}, D_t^{-\nu}, d_i^{-\nu}$ variables from aspiration. Thus, an IFG's transfer can be stated mathematically as follows:

$$\begin{aligned} &\mu_t(z_t(\bar{y},r)) + D_t^{-\mu} - D_t^{+\mu} = 1;\ \nu_t(z_t(\bar{y},r)) + D_t^{-\nu} - D_t^{+\nu} = 0, \forall t = 1,2,3,\ldots,T \\ &\mu_i(g_i(\bar{y},r)) + d_i^{-\mu} - d_i^{+\mu} = 1;\ \nu_i(g_i(\bar{y},r)) + d_i^{-\nu} - d_i^{+\nu} = 0, \forall i = 1,2,3,\ldots,p \end{aligned} \tag{14}$$

Since over-achievement, $D_t^{+\mu}, d_i^{+\mu}$ and under-achievement, $D_t^{-\nu}, d_i^{-\nu}$ from intuitionistic fuzzy goals (IFGs) provide full satisfaction and dissatisfaction for DM goals, those variables are removed from the Eqs. (14), Rukmani and Porchelvi (2018) [24]. Therefore, the above Eqs. (14) is condensed to:

$$\begin{aligned} &\mu_t(z_t(\bar{y},r)) + D_t^{-\mu} \geqslant 1;\ \nu_t(z_t(\bar{y},r)) - D_t^{+\nu} \leqslant 0,\ \forall t = 1,2,3,\ldots,T \\ &\mu_i(g_i(\bar{y},r)) + d_i^{-\mu} \geqslant 1;\ \nu_i(g_i(\bar{y},r)) - d_i^{+\nu} \leqslant 0,\ \forall i = 1,2,3\ldots,p \end{aligned} \tag{15}$$

We proposed an intuitionistic fuzzy non-dominant (IFND) solution for the developed MOLO (6) model or corresponding to IFMOLFO (3) problem based on goal programming Eqs. (15) and an intuitionistic fuzzy decision set. Thus, the proposed phase-I WIFGP model is summarized as:

Phase-I WIFGP Model:

$$\begin{aligned} &\min \sum_{t=1}^{T} \left(W_t^{\mu} D_t^{-\mu} + W_t^{\nu} D_t^{+\nu}\right) + \sum_{i=1}^{p} \left[w_i^{\mu} d_i^{-\mu} + w_i^{\nu} d_i^{+\nu}\right] \\ &\text{Subject to}: \left\{ \begin{array}{ll} \mu_t(z_t(\bar{y},r)) + D_t^{-\mu} \geqslant 1; & \nu_t(z_t(\bar{y},r)) - D_t^{+\nu} \leqslant 0, \forall t = 1,2,3,\ldots,T; \\ \mu_i(g_i(\bar{y},r)) + d_i^{-\mu} \geqslant 1; & \nu_i(g_i(\bar{y},r)) - d_i^{+\nu} \leqslant 0, \forall i = 1,2,3,\ldots,p \\ (\bar{y},r) \in S; & D_t^{-\mu}, D_t^{+\nu}, d_i^{-\mu}, d_i^{+\nu} \geqslant 0, \forall t,i \end{array} \right\} \end{aligned} \tag{16}$$

The numerical coefficients W_t^{μ}, w_i^{μ} and W_t^{ν}, w_i^{ν} in the proposed interactive Phase-I WIFGP (16) model describe the weight of relevance in achieving the intended levels for objective and constraint functions in the intuitionistic fuzzy environment, respectively. According to Mohammed's (1997) [23] weighting scheme, those weight of intuitionistic fuzzy goals are evaluated as follows:

$$W_t^{\mu} = \frac{1}{U_t^{\mu} - L_t^{\mu}} \text{ and } W_t^{\nu} = \frac{1}{U_t^{\nu} - L_t^{\nu}} \tag{17}$$

$$w_i^{\mu} = \frac{1}{q_i^{acc}} \text{ and } w_i^{\nu} = \frac{1}{q_i^{acc} - q_i^{rej}} \tag{18}$$

Definition 8. [28,14,13,30] Let $\bar{x}^* = \frac{\bar{y}^*}{r^*}$ where $(\bar{y}^*, r^*)$ be an optimal solution to proposed phase-I WIFGP (16) model or intuitionistic fuzzy non-dominant (IFND) solution to IFMOLFO (3), if there doesn't exists ($\nexists$) another $\bar{x} \in S^I$ such that $\mu_t(\tilde{z}_t^I(\bar{x})) \geqslant \mu_t(\tilde{z}_t^I(\bar{x}^*))$, and $\nu_t(\tilde{z}_t^I(\bar{x})) \leqslant \nu_t(\tilde{z}_t^I(\bar{x}^*))\, \forall t$, $\mu_i(\tilde{g}_i^I(\bar{x}^*)) \geqslant \mu_i(\tilde{g}_i^I(\bar{x}))$, and $\nu_i(\tilde{g}_i^I(\bar{x})) \leqslant \nu_i(\tilde{g}_i^I(\bar{x}^*)), \forall i$ and strictly inequality is hold for some t, i.

Theorem 4.1. *A unique intuitionistic fuzzy non-dominant (IFND) solution of the proposed phase-I WIFGP (16) model is also a Pareto-optimal solution to the IFMOLFO (3).*

Proof 3. Suppose $(\bar{y}^*, r^*)$ is unique IFND solution of proposed phase-I WIFGP (16) model. Then, $\mu_t(z_t(\bar{y}^*, r^*)) > \mu_t(z_t(\bar{y}, r))$ and $\nu_t(z_t(\bar{y}^*, r^*)) < \nu_t(z_t(\bar{y}, r))$ for all $t = 1, 2, 3, \ldots, T$. Assume that $(\bar{y}^*, r^*)$ is not Pareto-optimal solution to crisp MOLO (6). Then there exist $(\bar{y}, r) \in S$ such that $z_t(\bar{y}^*, r^*) \leqslant z_t(\bar{y}, r)\ \forall t$ and $z_t(\bar{y}^*, r^*) < z_t(\bar{y}, r)$ for some t. Then we have $z_t(\bar{y}^*, r^*) - L_t^\mu \leqslant z_t(\bar{y}, r) - L_t^\mu, \forall t$ and $z_t(\bar{y}^*, r^*) - L_t^\mu < z_t(\bar{y}, r) - L_t^\mu$,for some t. And $U_t^\nu - z_t(\bar{y}^*, r^*) \geqslant U_t^\nu - z_t(\bar{y}, r), \forall t$ and $U_t^\nu - z_t(\bar{y}^*, r^*) > U_t^\nu - z_t(\bar{y}, r)$, for some t. Since $U_t^\mu - L_t^\mu, U_t^\nu - L_t^\nu > 0$, we get $\frac{z_t(\bar{y}^*, r^*) - L_t^\mu}{U_t^\mu - L_t^\mu} \leqslant \frac{z_t(\bar{y}, r) - L_t^\mu}{U_t^\mu - L_t^\mu}, \forall t$and $\frac{z_t(\bar{y}^*, r^*) - L_t^\mu}{U_t^\mu - L_t^\mu} < \frac{z_t(\bar{y}, r) - L_t^\mu}{U_t^\mu - L_t^\mu}$, for some t. Also, $\frac{U_t^\nu - z_t(\bar{y}^*, r^*)}{U_t^\nu - L_t^\nu} \geqslant \frac{U_t^\nu - z_t(\bar{y}, r)}{U_t^\nu - L_t^\nu}, \forall t$ and $\frac{U_t^\nu - z_t(\bar{y}^*, r^*)}{U_t^\nu - L_t^\nu} > \frac{U_t^\nu - z_t(\bar{y}, r)}{U_t^\nu - L_t^\nu}$, for some t. This implies that $\mu_t(z_t(\bar{y}^*, r^*)) \leqslant \mu_t(z_t(\bar{y}, r)), \forall t$ and $\mu_t(z_t(\bar{y}^*, r^*)) < \mu_t(z_t(\bar{y}, r))$, for some t. and similarly we get $\nu_t(z_t(\bar{y}^*, r^*)) \geqslant \nu_t(z_t(\bar{y}, r)), \forall t$ and $\nu_t(z_t(\bar{y}^*, r^*)) > \nu_t(z_t(\bar{y}, r))$, for some t. This indicate that $(\bar{y}, r)$ is also an IFND solution to model (16). Hence, we attained at contradiction to $(\bar{y}^*, r^*)$ is unique IFND solution to Phase-I WIFGP (16) model. Therefore, using theorem-3.1 and 3.2, $\bar{x}^* = \frac{\bar{y}^*}{r^*}$ is Pareto-optimal solution to IFMOLFO (3). □

According to the theorem-4.1 if the uniqueness of the optimal solution $(\bar{y}^*, r^*)$ to Phase-I WIFGP (16) model is not guaranteed, it is not necessary to perform a Pareto-optimal test for $(\bar{y}^*, r^*)$. However, at least one of the multiple optimal solutions is a Pareto-optimal solution to the model. To find/select a Pareto-optimal solution from multiple optimal solutions, several models have been introduced in the literature. Most of the existing models were formulated in the fuzzy decision environment by Guu and Wu (1999) [16], Dubois and Fortemps (1999) [12], Jimenez and Bilbao (2009) [28]. But, in a fuzzy decision environment, it does not consider the degree of dissatisfaction and neutrality separately, which are key for real-life decision-making processes. Hence, to find a Pareto-optimal solution in the intuitionistic fuzzy environment, we have re-formulated a new model and consequently introduced a novel two-phase weighted intuitionistic fuzzy goal programming (WIFGP) method to obtain a compromise solution that satisfies both the intuitionistic fuzzy non-dominant and the Pareto-optimal solution.

Furthermore, in the case of at least one of the values of deviation variables $D_t^{-\mu}$ or $d_i^{-\mu}$ zero in the intuitionistic fuzzy non-dominant (IFND) solution, $(\bar{y}^*, r^*)$ to Phase-I WIFGP (16) model, this solution may not be secured for Pareto optimal solutions. To clarify this issue, let $(\bar{y}^*, r^*)$is an IFND solution to the multi-objective linear optimization (MOLO) model with $D_l^{-\mu^*} = 0$ for some $l \in \{1, 2, 3, \ldots, T\}$. This implies that $\mu_l(z_l(\bar{y}^*, r^*)) = 1$ (see Phase-I WIFGP (16) model) for the corresponding objective function value $z_l(\bar{y}^*, r^*)$. According to the definition of the membership function, $\mu_t(z_t(\bar{y}^*, r^*)) = 1$ for any $z_t(\bar{y}^*, r^*) \geqslant U_t^\mu$. For instance; if there exist a feasible point $(\bar{x}, s)$ such that $z_l(\bar{x}, s) > z_l(\bar{y}^*, r^*) \geqslant U_l^\mu$ for some $l \in \{1, 2, 3, \ldots, T\}$ and $z_t(\bar{x}, s) \geqslant z_t(\bar{y}^*, r^*), \forall t \neq l$, then $(\bar{x}, s)$ is Pareto dominant to an IFND solution $(\bar{y}^*, r^*)$ (similarly true for intuitionistic fuzzy constraint $g_i(\bar{y}, r)$ if for some $i, d_i^{-\mu^*} = 0$). Hence, an IFND solution $(\bar{y}^*, r^*)$ is not a Pareto-optimal solution to the MOLO model.

Therefore, to overcome the challenge of obtaining a Pareto-optimal solution to the IFMOLFO (3) problem, we proposed the following second phase WIFGP model by extending the approach suggested by Jimenez and Bibao (2009) [28] into an intuitionistic fuzzy decision environment and without losing the previous optimal solution in total.

Phase-II WIFGP Model:

$$\max \sum_{t\in F}^{T} w_t^z e_t^z + \sum_{i\in C}^{p} w_i^g e_i^g$$
$$\text{Subject to}: \left\{ \begin{array}{l} z_t(\bar{y}, r) - e_t^z = z_t(\bar{y}^{**}, r^{**});\ \forall t \in F; \\ g_i(\bar{y}, r) - e_i^g = g_i(\bar{y}^{**}, r^{**});\ \forall i \in C \cap i \in \{1, 2, 3, \ldots, k\} \\ g_i(\bar{y}, r) + e_i^g = g_i(\bar{y}^{**}, r^{**});\ \forall i \in C \cap i \in \{k+1, k+2, \ldots, p\} \\ \mu_t(z_t(\bar{y}, r)) + D_t^{-\mu} \geqslant 1;\ \nu_t(z_t(\bar{y}, r)) - D_t^{+\nu} \leqslant 0, \forall t \in G; \\ \mu_i(g_i(\bar{y}, r)) + d_i^{-\mu} \geqslant 1;\ \nu_i(g_i(\bar{y}, r)) - d_i^{+\nu} \leqslant 0, \forall i \in K \\ D_t^{-\mu} \leqslant D_t^{-\mu^{**}}, D_t^{+\nu} \leqslant D_t^{+\nu^{**}}\ \forall t \in G;\ d_i^{-\mu} \leqslant d_i^{-\mu^{**}}, d_i^{+\nu} \leqslant d_i^{+\nu^{**}}, \forall i \in K \\ (\bar{y}, r) \in S; D_t^{-\mu}, D_t^{+\nu}, d_i^{-\mu}, d_i^{+\nu}, e_t^z, e_i^g \geqslant 0, \forall t, \forall i. \end{array} \right\} \quad (19)$$

where; $(\bar{y}^{**}, r^{**})$ is an intuitionistic fuzzy non-dominant (IFND) solution to Phase-I WIFGP (16) model, with corresponding deviation variables $\left(D_t^{-\mu^{**}}, D_t^{+\nu^{**}}, d_i^{-\mu^{**}}, d_i^{+\nu^{**}}\right)$. Let us categorize an objective function $z_t(\bar{y}, r)$ as $F = \left\{t = 1, 2, 3, \ldots, T | D_t^{-\mu^{**}} = 0\right\}$,and determine its relative weight using $w_t^z = \frac{1}{|(z_t(\bar{y}^{**}, r^{**}))\left(U_t^\mu - L_t^\mu\right)|}$, $t \in F \& z_t(\bar{y}^{**}, r^{**}) \neq 0$. The other objective functions collected in set G, i.e., $G = \left\{1, 2, 3, \ldots, T | D_t^{-\mu^{**}} \neq 0\right\}$. Let us identify the constraint functions $g_i(\bar{y}, r)$ at IFND solution; as $C = \left\{i = 1, 2, 3, \ldots, p | d_i^{-\mu^{**}} = 0\right\}$, and calculate its relative weight by using $w_i^g = \frac{1}{|(g_i(\bar{y}^{**}, r^{**}))\left(q_i^{acc}\right)|}$, $i \in C \& g_i(\bar{y}^{**}, r^{**}) \neq 0$. The other constraint functions collected in set $K = \left\{i = 1, 2, 3, \ldots, p | d_i^{-\mu^{**}} \neq 0\right\}$. e_t^z and e_i^g are surplus (non-negative deviation) variables of the t^{th} objective and i^{th} constraint functions from $z_t(\bar{y}^{**}, r^{**})$ and $g_i(\bar{y}^{**}, r^{**})$, respectively.

Theorem 4.2. *Assume that* $(\bar{y}^*, r^*)$ *is an optimal solution to the phase-II WIFGP (19) model. Then,* $(\bar{y}^*, r^*)$ *is a Pareto-optimal solution to the IFMOLFO (3).*

Proof 4. Given that $(\bar{y}^*, r^*)$ is an optimal solution, then based on the constraints of the Phase-II WIFGP (19) model, this solution is an IFND solution to the IFMOLFO (3). Suppose that $(\bar{y}^*, r^*)$ is not a Pareto-optimal solution to the IFMOLFO (3) then based on the definition-7 there exists another IFND solution, $(\bar{x}, r)$ such that $z_t(\bar{y}^*, r^*) \leqslant z_t(\bar{x}, r), \forall t$ and $z_j(\bar{y}^*, r^*) < z_j(\bar{x}, r)$ for some j. Suppose that the subscript $j \in G$, then $\mu_t(z_t(\bar{y}^*, r^*)) \leqslant \mu_t(z_t(\bar{x}, r)), \forall t$ and $\mu_j(z_j(\bar{y}^*, r^*)) < \mu_j(z_j(\bar{x}, r))$, for some j. and similarly we get $\nu_t(z_t(\bar{y}^*, r^*)) \geqslant \nu_t(z_t(\bar{x}, r)), \forall t$ and $\nu_j(z_j(\bar{y}^*, r^*)) > \nu_j(z_j(\bar{x}, r))$, for some j. This is contradiction to $(\bar{y}^*, r^*)$ is an IFND solution. In other words, the only possibility of the subscript is $j \in F$, then $e^z_j(\bar{y}^*, r^*) < e^z_j(\bar{x}, r)$ and we have $(\bar{x}, r)$ is an IFND solution. Therefore, the following two inequalities hold: $\sum_{t\in F}^{T} w^z_t e^z_t(\bar{y}^*, r^*) < \sum_{t\in F}^{T} w^z_t e^z_t(\bar{x}, r)$ and $\sum_{i\in C}^{p} w^g_i e^g_i(\bar{y}^*, r^*) \leqslant \sum_{i\in C}^{p} w^g_i e^g_i(\bar{x}, r)$. Following this inequality, we obtain: $\sum_{t\in F}^{T} w^z_t e^z_t(\bar{y}^*, r^*) + \sum_{i\in C}^{p} w^g_i e^g_i(\bar{y}^*, r^*) < \sum_{t\in F}^{T} w^z_t e^z_t(\bar{x}, r) + \sum_{i\in C}^{p} w^g_i e^g_i(\bar{x}, r)$. This show that $(\bar{y}^*, r^*)$ is not an optimal solution to phase-II WIFGP (19) model, a contradiction. □

The models formulated in Phase-I WIFGP (16) and Phase-II WIFGP (19) can be combined into a novel two-phase weighted intuitionistic fuzzy goal programming (WIFGP) model for finding Pareto-optimal solutions to the IFMOLFO (3) problem.

5. Proposed solution approach, algorithm

Based on the discussion given above, we introduced an algorithm for the proposed approach for finding a Pareto-optimal solution to the intuitionistic fuzzy multi objective linear fractional optimization (IFMOLFO) problem. The step-wise solution approach/ algorithm is illustrated in the following ways:

Algorithm: Obtain Pareto-optimal solution to IFMOLFO problem.

Input: Decision-makers (DMs) set small positive tolerance $\gamma > 0$.
Output: Pareto-optimal solution, $\bar{x}^*$, to IFMOLFO problem.

Initial Steps:

Step 1: Formulate an IFMOLFO (3) problem.
Step 2: Obtain the crisp MOLFO (4) of IFMOLFO (3) problem through accuracy function (A^λ).
Step 3: Formulate the linear model of crisp MOLFO as shown in MOLO (6).
Step 4: Determine minimum, $Z^m_t = \min_{(\bar{y},r)\in S} z_t(\bar{y}, r)$ and maximum, $Z^M_t = \max_{(\bar{y},r)\in S} z_t(\bar{y}, r)$ value for each $z_t(\bar{y}, r), t = 1, 2, 3, \dots, T$ in the MOLO (6) model.
Step 5: Ask the DMs to set the aspiration levels based on step 4: tolerances, $(0 \leqslant q^{rej}_i < q^{acc}_i)$, upper, (U^μ_t, U^ν_t), and lower, (L^μ_t, L^ν_t), bounds for each intuitionistic fuzzy goals.

Main Steps:

Step 6: Construct the membership,$\mu_t(z_t(\bar{y}, r)), \mu_i(g_i(\bar{y}, r))$ and non-membership,$\nu_t(z_t(\bar{y}, r)), \nu_i(g_i(\bar{y}, r))$ functions as shown in Eqs. (9–13).
Step 7: Formulate a Phase-I WIFGP (16) model, solve it and obtain:$\bar{y}^{*1} \leftarrow (y_l, r, D^{-\mu}_t, d^{-\mu}_i, D^{+\nu}_t, d^{+\nu}_i)$
Step 8: IF ($\bar{y}^{*1}$ is not unique) OR ($D^{-\mu}_t = 0$ for some t) OR ($d^{-\mu}_i = 0$ for some i), THEN
Step 9: Formulate the Phase-II WIFGP (19) model, solve it and obtain: $\bar{y}^{*2}$ and calculate $\bar{x}^{*2} \leftarrow \frac{\bar{y}^{*2}}{r}$
ELSE, Calculate $\bar{x}^{*1} \leftarrow \frac{\bar{y}^{*1}}{r}$.
END IF
Step 10: IF ($\|z^I - z(\bar{x}^{*1})\|_2 \leqslant \gamma$) OR ($\|z^I - z(\bar{x}^{*2})\|_2 \leqslant \gamma$), THEN Stop, the DMs are satisfied.

Use the formula $\|z^I - z(\bar{x}^*)\|_2 = \frac{\sqrt{\sum_{t=1}^{T}\left(z^I_t - z_t(\bar{x}^*)\right)^2}}{2\times T}$, $z^I_t = \max_{\bar{x}\in S^c} z_t(\bar{x}), t = 1, 2, 3, \dots, T$

Assign $\bar{x}^* \leftarrow \bar{x}^{*1}$ or $\bar{x}^* \leftarrow \bar{x}^{*2}$. The current result, $\bar{x}^*$ is taken as Pareto-optimal solution.
Step 11: ELSE, Modify the value of parameters (some or all) in step 5 and go to step 6.
END IF
Output PRINT $\bar{x}^*$.

Definition 9. [11,32]. The distance metric function is given as follows in order to choose the preferred solution from among the other solutions already acquired using the various approaches described in the literature:

$$D(\bar{x}^*) = \|z^I - z(\bar{x}^*)\|_2 = \frac{\sqrt{\sum_{t=1}^{T}\left(z_t^I - z_t(\bar{x}^*)\right)^2}}{2\times T} \tag{20}$$

Where $z_t^I = \max_{\bar{x}\in S^c} z_t(\bar{x}),\ t = 1,2,3,\ldots,T$ (z_t^I is ideal/best individual value) and $\bar{x}^*$ is an optimal solution obtained from available methods in the literature. The minimum value of $D(\bar{x}^*)$ is indicates the most preferable solution among alternative is $\bar{x}^* \in S^c$.

6. Numerical illustration

In this section, we presented the real-life application problem and a numerical example from the literature in order to illustrate the efficiency and applicability of the proposed method.

6.1. Application: agricultural land allocation problem (ALAP)

Let us choose the most common k crop in the study area, with average production per unit area of cultivated land denoted by P_{cs} for each crop $c = 1,2,3,\ldots,k$ in season $s = 1,2,3,\ldots,l$. The decision variables x_{cs} used to represent the area of cultivation land for those crops c in season s and total area of cultivation land available for season $s = 1,2,3,\ldots,l$ is represented by AL_s. Let N_{cs}, Q_{cs}, and R_{cs}, represent the net profit gained, average cost of production, and labor requirement per unit area of cultivated land for each crop $c = 1,2,3,\ldots,k$ in season $s = 1,2,3,\ldots,l$, respectively. One of the key goals in the study area is achieving the minimum household essential food crop production requirement. Let $\overline{P}_{es}$ represent the minimum essential food crop production requirement in season s. In addition, TR_s used to represent the total labor required for this season s. The mathematical model of the multi-objective fractional agricultural land allocation problem (MOFALAP) in the intuitionistic fuzzy environment for crop $c = 1,2,3,\ldots,k$ with season $s = 1,2,3,\ldots,l$ is given as follows:

$$\begin{aligned}
&\max \tilde{z}_1^I(\bar{x}) = \frac{\tilde{N}_1^I(\bar{x})}{\tilde{D}_1^I(\bar{x})} = \frac{\sum_{s=1}^{l}\sum_{c=1}^{k}\tilde{N}_{cs}^I x_{cs} \rightarrow \text{Profit goal}}{\sum_{s=1}^{l}\sum_{c=1}^{k}\tilde{Q}_{cs}^I x_{cs} \rightarrow \text{Cost of production goal}}\\
&\max \tilde{z}_s^I(\bar{x}) = \frac{\tilde{N}_s^I(\bar{x})}{\tilde{D}_s^I(\bar{x})} = \frac{\sum_{c=1}^{k}\tilde{P}_{cs}^I x_{cs} \rightarrow \text{Crop production goal in season s}}{\sum_{c=1}^{k} x_{cs} \rightarrow \text{Land utilization goal in season s}}\\
&\text{Subject to :}\\
&\tilde{g}_s^I(\bar{x}) = \sum_{c=1}^{k}\tilde{R}_{cs}^I x_{cs} \tilde{\leqslant} \tilde{TR}_s^I,\ s = 1,2,3,\ldots,l \rightarrow \text{Labour requirement for crop c in season s}\\
&\tilde{g}_s^I(\bar{x}) = \sum_{e=1}^{n<k}\tilde{P}_{es}^I x_{es} \tilde{\geqslant} \tilde{\overline{P}}_{es}^I, e \in \{c\}, s = 1,2,3,\ldots,l \rightarrow \text{Family food requirement for crop e in season s}\\
&g_s(\bar{x}) = \sum_{c=1}^{k} x_{cs} \leqslant AL_s,\ s = 1,2,3,\ldots,l \rightarrow \text{Cultivable land availability for crop c in season s}\\
&x_{cs}, x_{es} \geqslant 0 \rightarrow \text{The area of the cultivated land for crop c in season s}
\end{aligned} \tag{21}$$

where, $\tilde{\leqslant}$, $\tilde{\geqslant}$ in the constraint used to represent the intuitionistic fuzzy inequality. The parameters, $\tilde{TR}_s^I, \tilde{P}_{cs}^I, \tilde{\overline{P}}_{es}^I, \tilde{N}_{cs}^I, \tilde{Q}_{cs}^I$ and $\tilde{R}_{cs}^I$ are triangular intuitionistic fuzzy numbers.

6.1.1. Illustration of the ALAP with case study

To illustrate the efficiency of the proposed approach, we have conducted a real-case study of an agricultural planning problem for farmers who are based on a small-scale irrigation system in Hawassa-Zuria, Sidama region, Ethiopia. The quantitative (numerical) data on crop production in quantal/hectare (qtl/ha), land use (ha), labor demand (man-days/ha), and cash (ETB) requirements for all crops over the course of the year was gathered from a variety of sources, including Hawassa agriculture research office reports, Ethiopian MoARD (2018) annual report [21], Meselu et.al. (2018) [20], FAO (2018) report [15], and current farming practices. The required data is summarized in Table 1.

In the dry season (locally Bega), the main crops cultivated in the study area are Cabbage (x_1), Onion (x_2), Potato (x_3), and Pepper (x_4), and in the semi-dry season (locally Belg), Maize (x_5), Bean (x_6), Sweet Potato (x_7), Carrot (x_8), and Tomato (x_9). The total area of land under cultivation is 3.5 hectares (ha) and is allocated for different crops for each Bega (December, January, February) and Belg (March, April, May) season in a year. The labour accessibility for each Bega and Belg season is almost given to be $\tilde{110}^I$ man-days. The existing framers need essentially at least $\tilde{45}^I$ quantal of potatoes in the Bega season while they need at least $\tilde{15}^I$ quantal of carrots in the Belg season to meet their annual food grain requirement for their basic needs.

Now, based on the given data as illustrated in Table 1 and model (21), the mathematical formulation of the given MOFALAP is summarized as follows:

Table 1
Source: Current farming practices,and unpublished office reports

Season	Crops	Production (qtl/ha) $\widetilde{P}^{I}_{cs}$	Net profit (ETB/ha.) $\widetilde{N}^{I}_{cs}$	Labor (man-day/ha) $\widetilde{R}^{I}_{cs}$	Cost (ETB/ha) $\widetilde{Q}^{I}_{cs}$
Bega	Cabbage	$\tilde{97.5}^{I}$	$\tilde{31070.21}^{I}$	$\tilde{75}^{I}$	$\tilde{12847.12}^{I}$
	Onion	$\tilde{115.55}^{I}$	$\tilde{71969}^{I}$	$\tilde{125}^{I}$	$\tilde{16520.2}^{I}$
	Potato	$\tilde{111.03}^{I}$	$\tilde{32350}^{I}$	$\tilde{119}^{I}$	$\tilde{14325}^{I}$
	Pepper	$\tilde{62.64}^{I}$	$\tilde{13241}^{I}$	$\tilde{60}^{I}$	$\tilde{10534.97}^{I}$
Belg	Maize	$\tilde{72.92}^{I}$	$\tilde{13500}^{I}$	$\tilde{89}^{I}$	$\tilde{10044}^{I}$
	Bean	$\tilde{37.04}^{I}$	$\tilde{14148.6}^{I}$	$\tilde{49}^{I}$	$\tilde{9500}^{I}$
	Sweet potato	$\tilde{84.76}^{I}$	$\tilde{39834.64}^{I}$	$\tilde{111}^{I}$	$\tilde{13254.02}^{I}$
	Carrot	$\tilde{44.85}^{I}$	$\tilde{13445}^{I}$	$\tilde{96}^{I}$	$\tilde{11265}^{I}$
	Tomato	$\tilde{61.81}^{I}$	$\tilde{31874}^{I}$	$\tilde{85}^{I}$	$\tilde{11780}^{I}$

$$
\begin{aligned}
&max\ \tilde{z}^{I}_{1}(\bar{x}) = \frac{\tilde{31070.21}^{I}x_1+\tilde{71969}^{I}x_2+\tilde{32350}^{I}x_3+\tilde{13241}^{I}x_4+\tilde{13500}^{I}x_5+\tilde{14148.6}^{I}x_6+}{\tilde{12847.12}^{I}x_1+\tilde{16520.2}^{I}x_2+\tilde{14325}^{I}x_3+\tilde{10534.97}^{I}x_4+\tilde{10044}^{I}x_5+\tilde{9500}^{I}x_6+}\\
&\frac{\tilde{39834.64}^{I}x_7+\tilde{13445}^{I}x_8+\tilde{31874}^{I}x_9 \rightarrow \text{Profit goal}}{\tilde{13254.02}^{I}x_7+11265x_8+\tilde{11780}^{I}x_9 \rightarrow \text{Cost of production goal}}\\
&max\ \tilde{z}^{I}_{2}(\bar{x}) = \frac{\tilde{97.5}^{I}x_1+\tilde{115.55}^{I}x_2+\tilde{111.03}^{I}x_3+\tilde{62.64}^{I}x_4 \rightarrow \text{Crop production goal in Bega season}}{x_1+x_2+x_3+x_4 \rightarrow \text{Land utilization goal in Bega season}}\\
&max\ \tilde{z}^{I}_{3}(\bar{x}) = \frac{\tilde{72.92}^{I}x_5+\tilde{37.04}^{I}x_6+\tilde{84.76}^{I}x_7+\tilde{44.85}^{I}x_8+\tilde{61.81}^{I}x_9 \rightarrow \text{Crop production goal in Belg season}}{x_5+x_6+x_7+x_8+x_9 \rightarrow \text{Land utilization goal in Belg season}}\\
&\text{Subject to :}\\
&\text{Essential food requirement constraints}\\
&\tilde{g}^{I}_{1}(\bar{x}) = \tilde{111.03}^{I}x_3 \tilde{\geqslant} \tilde{45}^{I} \rightarrow \text{Bega Season};\ \tilde{g}^{I}_{2}(\bar{x}) = \tilde{44.85}^{I}x_8 \tilde{\geqslant} \tilde{15}^{I} \rightarrow \text{Belg Season}\\
&\text{Labour requirement constraints}\\
&\tilde{g}^{I}_{3}(\bar{x}) = \tilde{75}^{I}x_1 + \tilde{125}^{I}x_2 + \tilde{119}^{I}x_3 + \tilde{60}^{I}x_4 \tilde{\leqslant} \tilde{110}^{I} \rightarrow \text{Bega Season}\\
&\tilde{g}^{I}_{4}(\bar{x}) = \tilde{89}^{I}x_5 + \tilde{49}^{I}x_6 + \tilde{111}^{I}x_7 + \tilde{96}^{I}x_8 + \tilde{85}^{I}x_9 \tilde{\leqslant} \tilde{110}^{I} \rightarrow \text{Belg Season}\\
&\text{Cultivate land availability constraints}\\
&g_5(\bar{x}) = x_1 + x_2 + x_3 + x_4 \leqslant 3.5 \rightarrow \text{Bega Season}\\
&g_6(\bar{x}) = x_5 + x_6 + x_7 + x_8 + x_9 \leqslant 3.5 \rightarrow \text{Belg Season}\\
&x_1, x_2, \ldots, x_9 \geqslant 0
\end{aligned} \tag{22}
$$

Where, all uncertain parameters set by DMs are symmetric triangular IFNs (see preposition 2.2).

Solution: Let the positive tolerance set by decision-makers (DMs) be $\gamma = 0.8506$. Firstly, we apply an accuracy ranking function A^{λ} on the given symmetric triangular IFNs and obtained crisp numbers. Thus, the crisp model of MOFALAP (22) summarized as:

$$
\begin{aligned}
&max\ z_1(\bar{x}) = \frac{31070.21x_1+71969x_2+32350x_3+13241x_4+13500x_5+14148.6x_6+}{12847.12x_1+16520.2x_2+14325x_3+10534.97x_4+10044x_5+9500x_6+}\\
&\frac{39834.64x_7+13445x_8+31874x_9}{13254.02x_7+11265x_8+11780x_9}\\
&max\ z_2(\bar{x}) = \frac{97.5x_1+115.55x_2+111.03x_3+62.64x_4}{x_1+x_2+x_3+x_4}\\
&max\ z_3(\bar{x}) = \frac{72.92x_5+37.04x_6+84.76x_7+44.85x_8+61.81x_9}{x_5+x_6+x_7+x_8+x_9}\\
&\text{Subject to :}\\
&111.03x_3 \tilde{\geqslant} 45, 44.85x_8 \tilde{\geqslant} 15, x_1 + x_2 + x_3 + x_4 \leqslant 3.5, x_5 + x_6 + x_7 + x_8 + x_9 \leqslant 3.5;\\
&75x_1 + 125x_2 + 119x_3 + 60x_4 \tilde{\leqslant} 310, 89x_5 + 49x_6 + 111x_7 + 96x_8 + 85x_9 \tilde{\leqslant} 310, x_1, x_2, \ldots, x_9 \geqslant 0
\end{aligned} \tag{23}
$$

Then, using the proposed variable transformation $\bar{y} = r\bar{x}$ the linear model of crisp MOFALAP (23) is formulated as:

$$
\begin{aligned}
&max\ z_1(\bar{y}, r) = 31070.21y_1 + 71969y_2 + 32350y_3 + 13241y_4 + 13500y_5 + 14148.6y_6 + 39834.64y_7\\
&+13445y_8 + 31874y_9\\
&max\ z_2(\bar{y}, r) = 97.5y_1 + 115.55y_2 + 111.03y_3 + 62.64y_4\\
&max\ z_3(\bar{y}, r) = 72.92y_5 + 37.04y_6 + 84.76y_7 + 44.85y_8 + 61.81y_9\\
&\text{Subject to :}\\
&y_1 + y_2 + y_3 + y_4 \leqslant 1; y_5 + y_6 + y_7 + y_8 + y_9 \leqslant 1\\
&12847.12y_1 + 16520.2y_2 + 14325y_3 + 10534.97y_4 + 10044y_5 + 9500y_6 + 13254.02y_7 + 11265y_8 + 11780y_9 \leqslant 1\\
&111.03y_3 - 45r \tilde{\geqslant} 0; 44.85y_8 - 15r \tilde{\geqslant} 0; y_1 + y_2 + y_3 + y_4 - 3.5r \leqslant 0;\\
&y_5 + y_6 + y_7 + y_8 + y_9 - 3.5r \leqslant 0; 75y_1 + 125y_2 + 119y_3 + 60y_4 - 110r \tilde{\leqslant} 0;\\
&89y_5 + 49y_6 + 111y_7 + 96y_8 + 85y_9 - 110r \tilde{\leqslant} 0; y_1, y_2, \ldots, y_9 \geqslant 0; r > 0.
\end{aligned} \tag{24}
$$

The upper, lower tolerance limits, and aspiration level of intuitionistic fuzzy goals for each objective function are then presented in Table 2 by decision-makers based on the individual optimal solutions of all objective functions under given constraints and pay-off table. In addition, decision-makers specify acceptance, q_i^{acc} and rejection, q_i^{rej}, where, $q_i^{rej} = 0.2 \times q_i^{acc}$ tolerance levels for each intuitionistic fuzzy constraint function in Table 3. Based on Table 2 and Table 3, we construct the membership and non-membership functions for both objective and constraint functions. Thus, the proposed phase-I WIFGP model is formulated as:

Phase-I WIFGP model:-

$$\begin{aligned}
&\min 1.1344D_1^{-\mu} + 1.2676D_1^{+\nu} + 374.98D_2^{-\mu} + 793.714D_2^{+\nu} + 633.78D_3^{-\mu} + 957.96D_3^{+\nu} + 0.4d_1^{-\mu} + \\
&0.5d_1^{+\nu} + 0.67d_2^{-\mu} + 0.833d_2^{+\nu} + 0.05d_3^{-\mu} + 0.0625d_3^{+\nu} + 0.033d_4^{-\mu} + 0.0417d_4^{+\nu} \\
&\text{Subject to}: \quad 12847.12y_1 + 15520.2y_2 + 14325y_3 + 10534.97y_4 + 10044y_5 + 9500y_6 + 13254.02y_7 \\
&+11265y_8 + 11780y_9 \le 1;\; y_1 + y_2 + y_3 + y_4 \le 1; \quad y_5 + y_6 + y_7 + y_8 + y_9 \le 1 \\
&y_1 + y_2 + y_3 + y_4 - 3.5r \le 0; \quad y_5 + y_6 + y_7 + y_8 + y_9 - 3.5r \leqslant 0 \\
&31070.21y_1 + 71969y_2 + 32350y_3 + 13241y_4 + 13500y_5 + 14148.6y_6 + 39834.64y_7 + 13445y_8 \\
&+31874y_9 + 0.88148887D_1^{-\mu} \geqslant 2.4837151 \\
&31070.21y_1 + 71969y_2 + 32350y_3 + 13241y_4 + 13500y_5 + 14148.6y_6 + 39834.64y_7 + 13445y_8 \\
&+31874y_9 + 0.7888928D_1^{+\nu} \geqslant 2.391119 \\
&97.5y_1 + 115.55y_2 + 111.03y_3 + 62.64y_4 + 0.0026668D_2^{-\mu} \geqslant 0.0051173 \\
&97.5y_1 + 115.55y_2 + 111.03y_3 + 62.64y_4 + 0.0012599D_2^{+\nu} \geqslant 0.004710448 \\
&72.92y_5 + 37.04y_6 + 84.76y_7 + 44.85y_8 + 61.81y_9 + 0.00157783D_3^{-\mu} \geqslant 0.00232209 \\
&72.92y_5 + 37.04y_6 + 84.76y_7 + 44.85y_8 + 61.81y_9 + 0.00104388D_3^{+\nu} \geqslant 0.00178814 \\
&111.03y_3 - 45r + 2.5d_1^{-\mu} \geqslant 0;\; 111.03y_3 - 45r + 2d_1^{+\nu} \geqslant -0.5;\; 44.85y_8 - 15r + 1.5d_2^{-\mu} \geqslant 0; \\
&44.85y_8 - 15r + 1.2d_2^{+\nu} \geqslant -0.3;\; -75y_1 - 125y_2 - 119y_3 - 60y_4 + 110r + 20d_3^{-\mu} \geqslant 0 \\
&75y_1 + 125y_2 + 119y_3 + 60y_4 - 110r - 16d_3^{-\nu} \leqslant 4;\; -89y_5 - 49y_6 - 111y_7 - 96y_8 - 85y_9 + 110r + 30d_4^{-\mu} \geqslant 0 \\
&89y_5 + 49y_6 + 111y_7 + 96y_8 + 85y_9 - 110r - 24d_4^{+\nu} \leqslant 6;\; y_1, y_2, \ldots, y_9 \geqslant 0; r > 0; \\
&D_t^{-\mu}, D_t^{+\nu} \geqslant 0, t = 1,2,3;\; d_i^{-\mu}, d_i^{+\nu} \geqslant 0, i = 1,2,3,4.
\end{aligned} \tag{25}$$

Applying the Lingo-19 software, for solving the above linear programming (LP) model (25) and the intuitionistic fuzzy non-dominant solutions are obtained as: $y_1 = 0, y_2 = 1.44012 \times 10^{-5}, y_3 = 3.110186\times\ 10^{-5}, y_4 = 0, y_5 = 3.050098 \times 10^{-5}, y_6 = 0, y_7 = 0, y_8 = 2.18413 \times 10^{-6}, r = 1.30088 \times 10^{-5}$ with corresponding deviation values $D_t^{-\mu^*} = 0,\ t = 1, 2,\ 3, d_1^{-\mu^*} = 0, D_t^{+\nu} = 0, \forall t; d_i^{-\nu} = 0, \forall i$ the others are non-zeros, $d_2^{-\mu} = 6.470332 \times 10^{-5}, d_3^{-\mu} = 2.035588 \times 10^{-4}, d_4^{-\mu} = 4.980558 \times 10^{-5}$. Now using back ward substitution $\bar{x} = \frac{\bar{y}}{r}, \left(x_l = \frac{y_l}{r}, l = 1, 2, \ldots, 9\right)$ we calculate the solutions to MOFALAP (22) as follows: $\bar{x}^{*1} = (x_1 = 0, x_2 = 1.107035, x_3 = 2.390832, x_4 = 0, x_5 = 2.344642, x_6 = 0, x_7 = 0, x_8 = 0.167896 \text{ and } x_9 = 0)$.

According to previous discussion, this solution may not be a Pareto-optimal solution for the MOFALAP (22), since one of the under-deviation variables is zero, as obtained in the intuitionistic fuzzy non-dominant solutions. Hence, we need to formulate the Phase-II WIFGP model as follows:

Table 2
Individual maximum $\left(Z_t^M\right)$, minimum $\left(Z_t^m\right)$ values, Upper, lower tolerance limits,and aspiration levels (goals) for each intuitionistic fuzzy objective function.

	Z_t^M	Z_t^m	U_t^μ	U_t^ν	$L_t^\mu = L_t^\nu$	Goals
$z_1(\bar{y}, r)$	4.637118	0	2.483715	1.891119	1.602226	2.483715
$z_2(\bar{y}, r)$	7.75079×10^{-3}	0	5.1173×10^{-3}	2.71045×10^{-3}	2.4505×10^{-3}	5.1173×10^{-3}
$z_3(\bar{y}, r)$	7.26006×10^{-3}	0	2.32209×10^{-3}	1.78814×10^{-3}	7.4426×10^{-4}	2.32256×10^{-3}

Table 3
Accepted and rejected tolerance limits for each intuitionistic fuzzy constraint function.

	$g_1(\bar{y}, r)$	$g_2(\bar{y}, r)$	$g_3(\bar{y}, r)$	$g_4(\bar{y}, r)$
Accepted tolerance limit $\left(q_i^{acc}\right)$	2.5	1.5	20	30
Rejection tolerance limit $\left(q_i^{rej}\right)$	0.5	0.3	4	6

Phase-II WIFGP Model:-

$$\begin{array}{l} \max 73276.93e_1^z + 0.4567e_2^z + 272935.19e_3^z + 0.001147e_1^g \\ \text{Subject to :} \\ 12847.12y_1 + 15520.2y_2 + 14325y_3 + 10534.97y_4 + 10044y_5 + 9500y_6 + 13254.02y_7 \\ +11265y_8 + 11780y_9 \le 1;\; y_1 + y_2 + y_3 + y_4 \le 1;\; y_5 + y_6 + y_7 + y_8 + y_9 \le 1; \\ y_1 + y_2 + y_3 + y_4 - 3.5r \leqslant 0;\; y_5 + y_6 + y_7 + y_8 + y_9 - 3.5r \leqslant 0 \\ 31070.21y_1 + 71969y_2 + 32350y_3 + 13241y_4 + 13500y_5 + 14148.6y_6 + 39834.64y_7 \\ +13445y_8 + 31874y_9 - e_1^z = 2.4837151;\; 97.5y_1 + 115.55y_2 + 111.03y_3 + 62.64y_4 - e_2^z = 0.0051173; \\ 72.92y_5 + 37.04y_6 + 84.76y_7 + 44.85y_8 + 61.81y_9 - e_3^z = 0.00232209;\; 111.03y_3 - 45r - e_4^g = 0; \\ 44.85y_8 - 15r + 1.5d_2^{-\mu} \geqslant 0;\; -89y_5 - 49y_6 - 111y_7 - 96y_8 - 85y_9 + 110r + 30d_4^{-\mu} \geqslant 0; \\ -75y_1 - 125y_2 - 119y_3 - 60y_4 + 110r + 20d_3^{-\mu} \geqslant 0;\; 75y_1 + 125y_2 + 119y_3 + 60y_4 - 110r - 16d_3^{-\nu} \leqslant 4; \\ 44.85y_8 - 15r + d_2^{+\nu} \geqslant -0.3;\; 89y_5 + 49y_6 + 111y_7 + 96y_8 + 85y_9 - 110r - 24d_4^{+\nu} \leqslant 6; \\ d_3^{-\mu} \leqslant 0.0002035588;\; d_2^{-\mu} \leqslant 0.00006740332;\; d_4^{-\mu} \leqslant 0.00004980558; \\ y_1, y_2, \ldots, y_9;\; r > 0;\; e_t^z, e_1^g \geqslant 0, t = 1,2,3;\; d_i^{-\mu}, d_i^{+\nu} \geqslant 0, i = 2,3,4. \end{array} \quad (26)$$

Using Lingo-19 software, we solve the above LP model (26) and the Pareto-optimal solutions are obtained as: $y_1 = 5.449275 \times 10^{-8}, y_2 = 1.514681 \times 10^{-5}, y_3 = 3.027806 \times 10^{-5}, y_4 = 0$
$y_5 = 3.055792 \times 10^{-5}, y_6 = 0, y_7 = 0, y_8 = 2.091562 \times 10^{-6}, y_9 = 0, r = 1.299410 \times 10^{-5}$ and using back ward substitution $\bar{x} = \frac{\bar{y}}{r}, (x_l = \frac{y_l}{r}, l = 1,2,\ldots,9)$ we calculate the Pareto-optimal solutions to MOFALAP (22) in the intuitionistic fuzzy decision environment as follows:

$\bar{x}^{*2} = (x_1 = 0.004193, x_2 = 1.1656683, x_3 = 2.330139, x_4 = 0, x_5 = 2.351677, x_6 = 0, x_7 = 0, x_8 = 0.160962, x_9 = 0)$. Since $\|z^l - z(\bar{x}^{*2})\|_2 \leqslant \gamma = 0.8506$, the decision-makers are satisfied with the current solution.

6.1.2. Analysis of the obtained results

The main question in the study area is "how to utilize the limited land resources for different crops to attain the least required consumption goals?" To answer this type of question, we developed a new effective mathematical method and applied it to solving a real-case study of the agricultural land allocation problem, which is a key subject in feeding a growing population with restricted resources. Thus, the obtained results for MOFALAP (22) describe the area of land (by hectare) allocated for each crop in order to optimize the agricultural land allocation planning model as follows:

The area of land allocated for cabbage, onion, potato, and pepper, respectively, are $x_1 = 0.004193, x_2 = 1.1656683, x_3 = 2.330139, x_4 = 0$ in the Bega (dry) season, for maize, bean, sweet potato, carrot, and tomato, respectively, are $x_5 = 2.351677, x_6 = 0, x_7 = 0, x_8 = 0.160962, x_9 = 0$ in the Belg (semi-dry) season. Furthermore, the optimal profit value obtained is 193,314.03 ETB/year[1], the cost of production is 78,123.663 ETB/year, crop production in Bega season is 393.817 quantal and in Belg season is 178.703 quantal, the land utilized in Bega is 3.5 hectares, and in Belg season is 2.513 hectares, with a degree of satisfaction of $\alpha = 1$ and dissatisfaction $\beta = 0$ for farmers (decision-makers).

As we observed the previous common agricultural land allocation plan of farmers in Hawassa-Zuria, Sidama region, Ethiopia is only considered to satisfy their basic needs for survival. Their annual aspiration level is, on average, 95,250.78 ETB in order to cover their fundamental necessities. However, the solutions produced by the suggested method are encouraging to satisfy the constraint functions and achieve the intuitionistic fuzzy goals as effectively as possible by utilizing the maximum cultivable land area that is readily available to farmers. Therefore, the suggested method can be utilized quickly and effectively to resolve real-crop planning problems.

6.1.3. Comparison

For sake of comparison, we consequently solved the MOFALAP (22) model using the available optimization methodology that is usually used by researchers to solve such type of problem. The most common suitable methodology registered in the literature of imprecise environment for solving this type of problems are intuitionistic fuzzy techniques suggested by Angelov (1997) [2] and fuzzy max–min approach introduced by Zimmermann (1978) [35]. The following results are obtained using those methodologies (The readers may need to see the Angelov(1997) [2] model (31) in Appendix-A and Zimmermann (1978) [35] model (32) in Appendix-B that developed for MOFALAP (22) model: $x_1 = 2.17642, x_2 = 0.776409, x_3 = 0.208268, x_4 = 0, x_5 = 1.996459, x_6 = 0, x_7 = 0, x_8 = 0.041792, x_9 = 0$. To measure the efficiency of solutions, distance function $D(\bar{x})$, degree of satisfaction, $\alpha \in [0,1]$ and dissatisfaction, $\beta \in [0,1]$ by DMs (farmers) are used for this problem. The minimum value of $D(\bar{x})$ & β and the maximum value of α are represents the higher performance of solution $\bar{x}$. From the results shown in Table 4, all these parameters are justified that, phase-II WIFGP (26) model generates a better optimal solution than any one of the phase-I WIFGP (25) model, the Angelov(1997) [2] (31) model and the Zimmermann (1978) [35] (32) model. Then, the phase-I WIFGP (26) model generates a better optimal solution than both the Angelov (1997) [2](31) model and

[1] ETB = Ethiopian Birr for symbol of money

Table 4
Comparison of results obtained from proposed Two-Phase WIFGP and the available methods in the literature.

	z_t^B	Phase-I WIFGP (25)	Phase-II WIFGP (26)	Angelov (31)	Zemmariam (32)
$z_1(\bar{x})$	3.3602	2.4485	2.4745	2.4536	2.4536
$z_2(\bar{x})$	113.9153	112.4605	112.5192	102.8261	102.8261
$z_3(\bar{x})$	75.533	71.0443	71.1218	72.3445	72.3445
α	1	1	1	0.998206	0.998206
β	0	0	0	0	0.001794
$D(\bar{x})$	0	0.800995	0.785114	0.929001	0.929001

Zimmermann (1978) [35] (32) model. Thus, our proposed Two-Phase WIFGP approach outperforms the Angelov(1997) [2] approach and Zimmermann [35] (1978) approach.

6.2. Numerical example

Consider the following crisp MOLFO problem presented by Borza & Rambely (2021) [7], Pei (2017) [25], Guzel (2013) [17] and Chakraborty & Gupta (2002) [9]:

$$\begin{aligned} &max\left(z_1(\bar{x}) = \tfrac{-3x_1+2x_2}{x_1+x_2+3}, z_2(\bar{x}) = \tfrac{7x_1+x_2}{5x_1+2x_2+1}\right) \\ &\text{Subject to}: \ \bar{x} \in S = \left\{ \bar{x} \in \Re^2 : -x_1 + x_2 \leqslant -1, \quad 2x_1 + 3x_2 \le 15, \quad -x_1 \leqslant -3, x_1, x_2 \geqslant 0 \right\} \end{aligned} \tag{27}$$

Solution: Here, observed that $z_1(\bar{x}) < 0, z_2(\bar{x}) > 0, \forall \bar{x} \in S$ and equivalent formulated as $\max_{\bar{x}\in S}\left(z_1(\bar{x}) = \frac{x_1+x_2+3}{3x_1-2x_2}, z_2(\bar{x}) = \frac{7x_1+x_2}{5x_1+2x_2+1}\right)$ [30,34]. Let the positive tolerance set by DM be $\gamma = 0.05801$.

The equivalent linear model of above MOLFO (27) problem is developed as:

$$\begin{aligned} &max\,(z_1(\bar{y}, r) = y_1 + y_2 + 3r, z_2(\bar{y}, r) = 7y_1 + y_2) \\ &\text{Subject to}: S^{yr} = \left\{ \begin{array}{ll} \tilde{g}_1^I(\bar{y}, r) = 3y_1 - 2y_2 \tilde{\leqslant} 1, & \tilde{g}_2^I(\bar{y}, r) = 3y_1 + 2y_2 + r \tilde{\leqslant} 1 \\ g_3(\bar{y}, r) = -y_1 + y_2 + r \le 0, & g_4(\bar{y}, r) = 2y_1 + 3y_2 - 15r \le 0 \\ g_5(\bar{y}, r) = -y_1 + 3r \le 0, & y_1, y_2 \geqslant 0, r > 0 \end{array} \right\} \end{aligned} \tag{28}$$

Let us consider the first two constraints of model (28) as intuitionistic fuzzy inequality, $\tilde{g}_1^I(\bar{y}, r), \tilde{g}_2^I(\bar{y}, r)$ with acceptance $q_1^{acc} = 5, q_2^{acc} = 24$ and rejection $q_1^{rej} = 1.2, q_2^{rej} = 5.8$ tolerance value and others constraints are crisp inequality. Thus, weighted value of intuitionistic fuzzy inequality constraints are $w_1^{\mu} = \frac{1}{q_1^{acc}} = 0.2, w_1^{\nu} = \frac{1}{q_1^{acc} - q_1^{rej}} = 0.263, w_2^{\mu} = \frac{1}{q_2^{acc}} = 0.042$ and $w_2^{\nu} = \frac{1}{q_2^{acc} - q_2^{rej}} = 0.055$. The individual maximum $\left(Z_t^M = \max_{(\bar{y},r)\in S^{yr}} z_t(\bar{y}, r)\right)$, minimum $\left(Z_t^m = \min_{(\bar{y},r)\in S^{yr}} z_t(\bar{y}, r)\right)$, upper (U_t^{μ}, U_t^{ν}), lower (L_t^{μ}, L_t^{ν}) tolerance limit, and weighted (W_t^{μ}, W_t^{ν}) value of intuitionistic fuzzy goal (IFG) for each linear objective function, $z_t(\bar{y}, r), t = 1, 2$ presented in Table 5. Therefore, based on these parameters the proposed phase-I WIFGP model of MOLFO (27) problem is formulated as:

$$\begin{aligned} &min\ 0.883D_1^{-\mu} + 7.519D_1^{+\nu} + 0.267D_2^{-\mu} + 0.364D_2^{+\nu} + 0.2d_1^{-\mu} + 0.263d_1^{+\nu} + 0.042d_2^{-\mu} + 0.055d_2^{+\nu} \\ &\text{Subject to}: \left\{ \begin{array}{ll} y_1 + y_2 + 3r + 1.133D_1^{-\mu} \geqslant 1.733, & -y_1 - y_2 - 3r - 0.133D_1^{+\nu} \leqslant -0.733 \\ 7y_1 + y_2 + 3.75D_2^{-\mu} \geqslant 5.85, & -7y_1 - y_2 - 2.75D_2^{+\nu} \leqslant -4.85 \\ -3y_1 + 2y_2 + 5d_1^{-\mu} \geqslant -6, & 3y_1 - 2y_2 - 3.8d_1^{+\nu} \le 1.2 \\ -5y_1 - 2y_2 - r + 24d_2^{-\mu} \geqslant -25, & 5y_1 + 2y_2 + r - 18.2d_2^{+\nu} \le 5.8 \\ -y_1 + y_2 + r \le 0, D_1^{-\mu}, D_1^{+\nu} \geqslant 0, & 2y_1 + 3y_2 - 15r \le 0, D_2^{-\mu}, D_2^{+\nu} \geqslant 0 \\ -y_1 + 3r \le 0, d_1^{-\mu}, d_1^{+\nu} \geqslant 0, & y_1, y_2, d_2^{-\mu}, d_2^{+\nu} \geqslant 0, r > 0 \end{array} \right. \end{aligned} \tag{29}$$

Solving the above linear programming (LP) (29) problem by Lingo-19 software, the IFND solution obtained is $y_1 = 0.758823, y_2 = 0.538235, r = 0.220588$ with all deviation variable values being zeros. And then applying the backward substitution, $\bar{x} = \frac{\bar{y}}{r}\left(x_l = \frac{y_l}{r}, l = 1, 2\right)$, to find the corresponding optimal solution for MOLFO (27) problem as $x_1 = 3.4400, x_2 = 2.4400$.Since one of under-deviation variables is zero, we need to formulate the phase-II WIFGP model of the MOLFO (27) problem as:

Table 5
Individual maximum $\left(Z_t^M\right)$, minimum $\left(Z_t^m\right)$ values, Upper, lower tolerance limits and weights $\left(W_t^{\mu}, W_t^{\nu}\right)$ for each objective function.

	Z_t^M	Z_t^m	U_t^{μ}	U_t^{ν}	$L_t^{\mu} = L_t^{\nu}$	W_t^{μ}	W_t^{ν}
$z_1(\bar{y}, r)$	9.733	0.6	1.733	0.733	0.6	0.883	7.519
$z_2(\bar{y}, r)$	28.85	2.1	5.85	4.85	2.1	0.267	0.364

Table 6
Comparison of the results obtained from the proposed Two-Phase WIFGP and the available methods in the literature.

$\tilde{x}_1, \tilde{x}_2, \hat{x}_3, \hat{x}_4, \bar{x}^{1*}, \bar{x}^{2*}$	$z_1(\bar{x})$	$z_2(\bar{x})$	$D(\bar{x})$
$z_t^I = \max_{\bar{x}\in S} z_t(\bar{x}),\ t = 1, 2$	−0.609	1.364	0
Zimmermann (1978) [35] approach ($\tilde{x}_1$)	−1.073	1.23	0.12074
Angelov(1997) [2] approach ($\tilde{x}_2$)	−0.716	1.17	0.05539
Borza & Rambely (2021) [7] approach ($\hat{x}_3$)	−0.9547	1.2137	0.09424
Linearization fuzzy approach [25,9] ($\hat{x}_4$)	−0.625	1.15	0.05365
Phase-I WIFGP model ($\bar{x}^{*1}$)	−0.613	1.15	0.05351
Phase-II WIFGP model ($\bar{x}^{*2}$)	−0.609	1.15	0.05350

$$\max \quad 0.45e_1^z + 1.285e_2^z + 0.167e_1^g + 0.0082e_2^g$$
$$\text{Subject to}: \left\{ \begin{array}{ll} y_1 + y_2 + 3r - e_1^z = 1.959, & 7y_1 + y_2 - e_2^z = 5.85 \\ 3y_1 - 2y_2 + e_1^g = 1.2, & 5y_1 + 2y_2 + r + e_2^g = 5.8 \\ -y_1 + y_2 + r \leq 0, -y_1 + 3r \leq 0, & 2y_1 + 3y_2 - 15r \leq 0 \\ e_1^z, e_2^z, e_1^g, e_2^g \geqslant 0, & y_1, y_2 \geqslant 0, r > 0 \end{array} \right\} \tag{30}$$

Solving the above LP(30) problem, we obtained the optimal-solution $y_1 = 0.7714286, y_2 = 0.5571429, r = 0.2142857$ and then Pareto-optimal solution of MOLFO (27) problem is $\bar{x}^{*2} = (x_1 = 3.60, x_2 = 2.60)$. Since $\|z^I - z(\bar{x}^{*2})\|_2 \leqslant \gamma = 0.05801$, the decision-makers are satisfied with the current solution.

6.2.1. Comparison

The solution obtained by Borza & Rambely (2021) [7] fuzzy transformation approach is $\hat{x}_3 = (x_1 = 3, x_2 = 1.1073)$, Pei (2017) [25], and Chakraborty & Gupta (2002) [9] linearization fuzzy approach is $\hat{x}_4 = (x_1 = 3, x_2 = 2)$, proposed phase-I WIFGP model is $\bar{x}^{*1} = (x_1 = 3.44, x_2 = 2.44)$ and proposed phase-II WIFGP model is $\bar{x}^{*2} = (x_1 = 3.6, x_2 = 2.6)$. Additionally, solving the given MOLFO (27) problem by Zimmermann (1978) [35] approach and Angelov(1997) [2] approach using the same parameters presented in Table 5, we obtained the solutions $\tilde{x}_1 = (x_1 = 3.6, x_2 = 1.21)$ and $\tilde{x}_2 = (x_1 = 3.84, x_2 = 2.44)$, respectively. To compare the efficiency of the obtained results, we used the distance metric function $D(\bar{x})$ (see Eqs. 20) as suggested by [11,13]. As we observed from Table 6, we have

$$D(\bar{x}^{*2}) < D(\bar{x}^{*1}) < D(\hat{x}_4) < D(\tilde{x}_2) < D(\hat{x}_3) < D(\tilde{x}_1)$$

where, $\hat{x}_3, \hat{x}_4, \bar{x}^{*1}, \bar{x}^{*2}, \tilde{x}_1$ and $\tilde{x}_2$ represent the solution generated by Borza & Rambely (2021) [7] approach, linearization fuzzy approach [25,9], proposed phase-I WIFGP, phase-II WIFGP model, Zimmermann (1978) [35] approach and Angelov(1997) [2] approach, respectively. Thus, the proposed phase-II WIFGP model offers the most preferable Pareto-optimal solution than phase-I WIFGP model, Borza & Rambely (2021) [7] approach, linearization fuzzy approach suggested by [25,9], Zimmermann (1978) [35] approach and Angelov(1997) [2] approach. Then the solution obtained from the proposed phase-I WIFGP model is preferable to other approaches. Therefore, even if the proposed phase-I WIFG model, the fuzzy transformation approach of Borza and Rambely (2021) [7], the linearization fuzzy approach of Pei (2017) [25] and Chakraborty & Gupta (2002) [9], the Zimmermann (1978) [35] fuzzy max–min approach, and the Angelov (1997) [2] intuitionistic fuzzy optimization approach produce results that satisfy the condition of a fuzzy or intuitionistic fuzzy non-dominant solution, the solution is not always guaranteed to be Pareto-optimal for the MOLFO (27) problem.

7. Conclusion

In this paper, a new approach has been introduced for handling intuitionistic fuzzy multi-objective linear fractional optimization (IFMOLFO) problems. Due to uncontrollable factors, all uncertain parameters in IFMOLFO problems are set as triangular intuitionistic fuzzy numbers and the crisp MOLFO problem is obtained utilizing the accuracy ranking functions method. Then, using variable transformations, the problem is converted into a crisp multi-objective linear optimization problem, and the resultant problem is transformed into a single objective linear programming problem by utilizing the WIFGP model and obtaining an intuitionistic fuzzy non-dominant solution to IFMOLFO problems. This model is considered a phase-I WIFGP model.

Furthermore, to generate a compromise solution that satisfies both intuitionistic fuzzy non-dominant and Pareto-optimal solutions, the proposed method develops a phase-II WIFGP model in the intuitionistic fuzzy environment. Finally, the IFMOLFO problem and Two-Phase WIFGP model were implemented in a real case study of an agricultural planning problem, numerical example from literature and the results were justified efficiently. The practical implications are shown to greatly aid researchers in applying the suggested method to a variety of multi-objective real-world optimization problems.

In the future, we plan to extend the proposed method to handle different non-linear and integer multi-objective application problems like inventory control problems.

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Demmelash Mollalign Moges: Conceptualization, Investigation, Formal analysis, Methodology, Visualization, Software, Data curation, Writing - original draft. **Allen Rangia Mushi:** Data curation, Validation, Supervision, Writing - review & editing. **Berhanu Guta Wordofa:** Resources, Software, Validation, Supervision, Project administration.

Data availability

No data was used for the research described in the article.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. The Angelov (1997) [2] model for MOALAP (24)

$$
\begin{aligned}
&\max \alpha - \beta \\
&\text{Subject to}: \\
&12847.12y_1 + 15520.2y_2 + 14325y_3 + 10534.97y_4 + 10044y_5 + 9500y_6 + 13254.02y_7 + 11265y_8 + 11780y_9 \le 1 \\
&y_1 + y_2 + y_3 + y_4 \le 1; y_5 + y_6 + y_7 + y_8 + y_9 \le 1 \\
&y_5 + y_6 + y_7 + y_8 + y_9 - 3.5r \leqslant 0; 31070.21y_1 + 71969y_2 + 32350y_3 + 13241y_4 + 13500y5+ \\
&14148.6y_6 + 39834.64y_7 + 13445y_8 + 31874y_9 - 0.88148887\alpha \geqslant 1.6022262; \\
&y_1 + y_2 + y_3 + y_4 - 3.5r \leqslant 0; 31070.21y_1 + 71969y_2 + 32350y_3 + 13241y_4 + 13500y_5+ \\
&14148.6y_6 + 39834.64y_7 + 13445y_8 + 31874y_9 + 0.7888928\beta \geqslant 1.891119 \\
&111.03y_3 - 45r - 2.5\alpha \geqslant -2.5; 97.5y_1 + 115.55y_2 + 111.03y_3 + 62.64y_4 - 0.0026668\alpha \geqslant 0.0024505; \\
&111.03y_3 - 45r + 2\beta \geqslant -0.5; 97.5y_1 + 115.55y_2 + 111.03y_3 + 62.64y_4 + 0.0012599\beta \geqslant 0.002710448; \\
&44.85y_8 - 15r - 1.5\alpha \geqslant -1.5; 72.92y_5 + 37.04y_6 + 84.76y_7 + 44.85y_8 + 61.81y_9 - 0.00157783\alpha \geqslant 0.00074426; \\
&44.85y_8 - 15r + 1.2\beta \geqslant -0.3; 72.92y_5 + 37.04y_6 + 84.76y_7 + 44.85y_8 + 61.81y_9 + 0.00104388\beta \geqslant 0.00178814; \\
&-75y_1 - 125y_2 - 119y_3 - 60y_4 + 110r - 20\alpha \geqslant -20; 75y_1 + 125y_2 + 119y_3 + 60y_4 - 110r - 16\beta \le 4; \\
&-89y_5 - 49y_6 - 111y_7 - 96y_8 - 85y_9 + 110r - 30\alpha \geqslant -30; \\
&89y_5 + 49y_6 + 111y_7 + 96y_8 + 85y_9 - 110r - 24\beta \leqslant 6; y_1, \ldots, y_9 \geqslant 0; r > 0; \\
&\beta - \alpha \leqslant 0; \alpha + \beta \le 1; \beta \geqslant 0;
\end{aligned}
\tag{31}
$$

Appendix B. The Zimmermann (1978) [35] model for MOALAP (24)

$$
\begin{aligned}
&\max \alpha \\
&\text{Subject to}: \\
&12847.12y_1 + 15520.2y_2 + 14325y_3 + 10534.97y_4 + 10044y_5 + 9500y_6 + 13254.02y_7 + 11265y_8 + 11780y_9 \le 1 \\
&y_1 + y_2 + y_3 + y_4 \le 1; y_5 + y_6 + y_7 + y_8 + y_9 \le 1 \\
&y_5 + y_6 + y_7 + y_8 + y_9 - 3.5r \leqslant 0; 31070.21y_1 + 71969y_2 + 32350y_3 + 13241y_4 + 13500y5+ \\
&14148.6y_6 + 39834.64y_7 + 13445y_8 + 31874y_9 - 0.88148887\alpha \geqslant 1.6022262; \\
&y_1 + y_2 + y_3 + y_4 - 3.5r \leqslant 0; 97.5y_1 + 115.55y_2 + 111.03y_3 + 62.64y_4 - 0.0026668\alpha \geqslant 0.0024505; \\
&111.03y_3 - 45r - 2.5\alpha \geqslant -2.5; 72.92y_5 + 37.04y_6 + 84.76y_7 + 44.85y_8 + 61.81y_9 - 0.00157783\alpha \geqslant 0.00074426; \\
&44.85y_8 - 15r - 1.5\alpha \geqslant -1.5; -75y_1 - 125y_2 - 119y_3 - 60y_4 + 110r - 20\alpha \geqslant -20; \\
&-89y_5 - 49y_6 - 111y_7 - 96y_8 - 85y_9 + 110r - 30\alpha \geqslant -30; r > 0; y_1, \ldots, y_9 \geqslant 0; 0 \leqslant \alpha \le 1
\end{aligned}
\tag{32}
$$

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