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Solving Multi-Objective Multilevel Programming problems using two-phase Intuitionistic Fuzzy Goal Programming method

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ABSTRACT

This paper presents a two-phase intuitionistic fuzzy goal programming (two-phase IFGP) algorithm to solve Multi-Objective Multilevel Programming (MO-MLP) problems. The coefficient of each objective and constraint function is assumed to be triangular intuitionistic fuzzy parameters and the crisp MO-MLP problems are obtained using the accuracy function method. To avoid decision lock, the top levels set tolerance limits for decision variables to control the lower levels. The problem is modeled in the intuitionistic fuzzy environment using membership and non-membership functions for each objective function at all levels and decision variables controlled by the top levels. Then, we proposed an IFGP algorithm to achieve the highest degree of each membership and non-membership goal by minimizing unwanted deviational variables and generating compensatory solutions for all decision-makers at all levels. Moreover, in the proposed approach, two-phase IFGP is modeled to yield a compromise solution that satisfies both the MN-Pareto optimal solution and the Pareto optimal solution at each level. Also, verification of the proposed method is discussed with numerical examples.

1. Introduction

Most existing mathematical programming formulations concentrate on the problem involving only a single decision-maker (DM) for the decision system rather than the cooperative decision process involving multiple decision-makers. However, many decision-making problems have a hierarchical decision-making structure, with each aim being self-determined and often conflicting. As a result, hierarchical decision-making issues are formulated as multilevel programming (MLP) problems. The basic concept behind the MLP methods is that a leader-level DM sets his or her targets or goals and then searches for optimal solutions from each subordinate level of the organization, which are calculated independently. The judgments of the follower-level DMs are then submitted and changed by the leader-level DM with the organization's overall benefit in mind [1–4]. Most often, real-life application problems need to optimize two or more objective functions at each level of decision-makers. Such kinds of hierarchical decision-making problems are known as multi-objective multilevel programming (MO-MLP) problems [1,5–9].

In the standard optimization model of MLP problems, it is assumed that the coefficient and aspiration level of the objective and constraint functions are crisp values. However, in many real-planning problems, this assumption is usually difficult to set precise values due to the existence of uncertainty and imprecise information in forming MLP problems [2,6,10]. In addition, even if the MLP problems have a crisp structure, a classical optimization model is not flexible due to its strict discrimination given between satisfactory and dissatisfactory solutions. This frequently results in a follower-level DM decision control dominating that of a leader-level DM, which is a contradiction to the power of the leader-level DM.

To overcome the limitations of the crisp optimization methods to solve MLP problems, Lai et al. (1996) [4], Sinha et al. (2003) [3], Abo-Sinna et al. (2004) [11] and Sakawa et al. (2000) [12] introduced a fuzzy approach, after Bellman and Zadeh (1970) [13] developed a fuzzy decision-making environment. Zimmermann (1978) [14] was the first to use fuzzy set theory to decision problems involving several competing goals. Following that, many variants of fuzzy programming were studied and widely disseminated in the literature [2,4,6,7,9,10,15–19]. Mohamed (1997) [17] introduced the fuzzy goal programming (FGP) approach to overcome the shortage of fuzzy approaches introduced by Shih et al. (2003) [3]. Recently, several researchers, Baky (2010) [1], Pramanik and Roy (2007) [10], Arora and Gupta (2009) [16], Lachhwani (2015) [9], Arbaiy and Watada (2012) [8],

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Peric et al. (2019) [18], and Osman et al. (2017 [6], 2016 [7]) are extending the FGP approach suggested by Mohamed (1997) [17] to solve different domains of MO-MLP problems under a fuzzy environment.

Even though the fuzzy type model is more flexible and richer in representing an MLP problem under an imprecise environment than the crisp type model, it has a limit in modeling a problem when the imprecise coefficient and aspiration-level of objective and constraint function provide some degree of hesitation Rukmani and Porchelvi (2018) [20], Razmi et al. (2016) [21], Atanassov (1986) [22]. Due to such a drawback, many researchers extended the regular fuzzy set of basic concepts. Out of many generalized regular fuzzy sets, intuitionistic fuzzy sets (IFSs) introduced by Atanassov (1986) [22] have been developed to be extremely important for dealing with uncertainty and vagueness. Anglove (1997) [23] introduced intuitionistic fuzzy optimization strategies to address multi-objective programming (MOP) problems in an imprecise environment by extending the decision-making strategy proposed by Bellman and Zadhe (1970) [13].

Various works associated with MOP problems have been studied in recent decades using the intuitionistic fuzzy goal programming (IFGP) approach Razmi et al. (2016) [21], Dey and Roy (2015) [24], Rukmani and Porchelvi (2018) [20], Singh and Yadav (2018) [25], Bharati and Singh (2019) [26], El Sayed and Abo-Sinna (2021) [27], Ahmadini and Ahmad (2021) [28], Ahmad (2021) [29]. Sharma et al. (2021) [5] used accuracy function ranking methods to defuzzify intuitionistic fuzzy numbers (IFNs) first, then solve Multi-Objective Bi-level Programming (MO-BLP) problems using a fuzzy approach and TOPSIS method. To the best of the author's knowledge in the problem domain, the intuitionistic fuzzy optimization of multiple objectives in multilevel has never been modeled to date. Thus, this leads us to demonstrate the current study, proving that the proposed two-phase intuitionistic fuzzy goal programming (IFGP) approach for MO-MLP problems can contribute to future research in the field of imprecise multilevel optimization.

Since our developed model of the MLP problem includes several objective functions at each level of decision-makers with intuitionistic fuzzy parameters, the proposed new method for solving the MO-MLP problem is considered the main contribution to this research work. Additionally, the following points can be considered significant contributions to this research work:

- Intuitionistic fuzzy optimization approaches are one of the most effective tools for modeling optimization problems in an imprecise environment, and they produce more pleasing outcomes than crisp and fuzzy optimization strategies. Therefore, the use of an intuitionistic fuzzy decision environment to handle uncertainties, an interactive goal programming approach to appreciate the learning process about the problem, and a phase-II IFGP model to find a Pareto optimal solution to the MOP problem at each level, all contribute to the strength of this proposed method.
- All uncertain parameter values of the MO-MLP problem have been considered as triangular IFNs and their corresponding crisp form was obtained through the accuracy function ranking method.
- The aspiration levels of each objective function and the decision variables controlled by leader-level DM are represented as intuitionistic fuzzy goals (IFGs) to avoid decision lock for lower levels in the intuitionistic fuzzy decision environments.
- The concepts of MN-Pareto and Pareto optimality for the solutions to the MO-MLP problem are briefly described in this study.
- The main challenge that occurred in the literature of the method that was used to solve the MO-MLP problem under an imprecise environment was that it generated an MN-Pareto optimal solution, and this solution was considered Pareto optimal. However, this is not always guaranteed due to the value of membership and non-membership functions used to measure the degree of satisfaction and dissatisfaction of each DM at all levels regarding achieving the aspired levels of IFGs being bounded between 0 and 1 only, which is not always mathematically or logically correct. To overcome this difficulty, the study developed a new phase-II IFGP model.
- The flexible normalized weighted scheme proposed by Mohamed (1997) [17] is extended in this study to measure the relative importance of IFGs appropriately. In this work, four types of numerical weights are used in the phase-I IFGP model.
- The well-known numerical examples and distance measurement function are used to illustrate and compare the proposed method with the existing suggested methodology for solving the MO-MLP problem. The proposed solution's efficiency was demonstrated using the distance measure function.

The following is a description of the paper's structure: Section 2 contains preliminaries, basic notions, and terminology related to intuitionistic fuzzy set theory. In Section 3, the intuitionistic fuzzy MO-MLP problem and its equivalent crisp model are mathematically stated. Section 4 introduces the proposed solution approach and algorithm for solving the MO-MLP problem. In Section 5, a numerical example is provided to clarify the established two-phase IFGP technique. The final section contains concluding and future remarks.

2. Preliminary basic concepts

In this section, some basic concepts and preliminary results used in this paper are briefly introduced [5,6,22].

Definition 2.1. Assume that X is the general set of object x. Then the set $\tilde{A}^I = \{(x, \mu_{\tilde{A}^I}(x), \nu_{\tilde{A}^I}(x)) : x \in X\}$ is an **intuitionistic fuzzy set (IFS)**, where, $\mu_{\tilde{A}^I} : X \to [0,1]$ and $\nu_{\tilde{A}^I} : X \to [0,1]$ represents the degree of belonging (membership) and degree of non-belonging (non-membership) of $x \in \tilde{A}^I$ such that $0 \leq \mu_{\tilde{A}^I}(x) + \nu_{\tilde{A}^I}(x) \leq 1$ for all x in X. For any IFS $\tilde{A}^I$ on X, $\pi_{\tilde{A}^I}(x) = 1 - \mu_{\tilde{A}^I}(x) - \nu_{\tilde{A}^I}(x)$ represents the degree of indeterminacy (hesitancy) for $x \in \tilde{A}^I$ or $x \notin \tilde{A}^I$

Definition 2.2. Triangular Intuitionistic Fuzzy Number (TrIFN) denoted by $\tilde{a}^I = \langle \underline{a}, a, \bar{a}; \underline{b}, a, \bar{b} \rangle$ where, $\underline{a}, a, \bar{a}, \underline{b}, \bar{b} \in \Re$ such that $\underline{b} \leq \underline{a} \leq a \leq \bar{a} \leq \bar{b}$ is a special IFS on the real number set $\Re$, may show an imprecise data like "nearly a", which is "approximately equal to a". That means, the most acceptable value is a.

The membership and non-membership functions of TrIFN $\tilde{a}^I = \langle \underline{a}, a, \bar{a}; \underline{b}, a, \bar{b} \rangle$ respectively defined as follows:

$$\mu_{\tilde{a}^I}(x) = \begin{cases} \frac{x-\underline{a}}{a-\underline{a}} & \text{if } \underline{a} \leq x < a \\ 1 & \text{if } x = a \\ \frac{\bar{a}-x}{\bar{a}-a} & \text{if } a < x \leq \bar{a} \\ 0 & \text{if } x < \underline{a}, x > \bar{a} \end{cases} \qquad \nu_{\tilde{a}^I}(x) = \begin{cases} \frac{a-x}{a-\underline{b}} & \text{if } \underline{b} \leq x < a \\ 0 & \text{if } x = a \\ \frac{x-a}{\bar{b}-a} & \text{if } a < x \leq \bar{b} \\ 1 & \text{if } x < \underline{b}, x > \bar{b} \end{cases} \tag{1}$$

Definition 2.3. Let an arbitrary triangular IFN($\Re$) given as $\tilde{a}^I = \langle(\underline{a}, a, \bar{a}; \underline{b}, a, \bar{b})\rangle$. Then the score function of $\tilde{a}^I$ with respect to the membership and non-membership function are denoted by $S^\mu(\tilde{a}^I)$ and $S^\nu(\tilde{a}^I)$ respectively were obtained as follows:

$$S^\mu(\tilde{a}^I) = \frac{\underline{a} + 2a + \bar{a}}{4} \text{ and } S^\nu(\tilde{a}^I) = \frac{\underline{b} + 2a + \bar{b}}{4} \tag{2}$$

The weighted average of score functions for TrIFN $\tilde{a}^I$, $S_\lambda(\tilde{a}^I) = (1-\lambda)S^\nu(\tilde{a}^I) + \lambda S^\mu(\tilde{a}^I)$ is the accuracy function of triangular IFN(R) $\tilde{a}$ where $\lambda \in [0,1]$ is a weight set by DM first.

3. The mathematical formulation of the problem

3.1. Problem definition

Assume that at each tth-level of MLP problems have more than one objective function to be maximized. Consider that the tth-level decision-maker (denoted by DM_t) control over a decision vector $\bar{x}_t = (x_{t1}, x_{t2}, \ldots x_{tn_t})' \in R^{n_t}$, $\forall t = 1, 2, \ldots, T$. In addition, assume that $Z_t(\bar{x}) =$

$Z_t(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_T) : \mathfrak{R}^{n_1} \times \mathfrak{R}^{n_2} \times \cdots \times \mathfrak{R}^{n_T} \rightarrow \mathfrak{R}^{p_t}$ $t = 1, 2, \ldots, T$ represents the well defined linear objective functions for $DM_t, t = 1, 2, \ldots, T$. The mathematical formulation of intuitionistic fuzzy Multi-Objective Multi-level Programming (MO-MLP) problem is defined as follows: [1,6,7,10]

$$\max_{\bar{x}_1} \tilde{Z}_1(\bar{x}) = \max_{\bar{x}_1}(\tilde{z}_{11}(\bar{x}), \tilde{z}_{12}(\bar{x}), \ldots, \tilde{z}_{1p_1}(\bar{x})) \qquad [\text{1st-}Level]$$

for given $\bar{x}_1$; where, $\bar{x}_2, \bar{x}_3, \ldots, \bar{x}_T$ solves

$$\max_{\bar{x}_2} \tilde{Z}_2(\bar{x}) = \max_{\bar{x}_2}(\tilde{z}_{21}(\bar{x}), \tilde{z}_{22}(\bar{x}), \ldots, \tilde{z}_{2p_2}(\bar{x})) \qquad [\text{2nd-}Level]$$

$$\vdots$$

for given $\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_{T-1}$; where, $\bar{x}_T$ solve

$$\max_{\bar{x}_T} \tilde{Z}_T(\bar{x}) = \max_{\bar{x}_T}(\tilde{z}_{T1}(\bar{x}), \tilde{z}_{T2}(\bar{x}), \ldots, \tilde{z}_{Tp_T}(\bar{x})) \qquad [T\text{th-}Level]$$

Subject to:

$$\bar{x} \in \tilde{S}^I = \{\bar{x} \in R^n : \tilde{A}_1^I \bar{x}_1 + \tilde{A}_2^I \bar{x}_2 + \cdots + \tilde{A}_T^I \bar{x}_T (\geq, \leq \ or \ =) \tilde{B}^I, \bar{x} \geq \bar{0}\} \tag{3}$$

Where,

- $\tilde{z}_{tj}(\bar{x}) j = 1, 2, \ldots, p_t$ is a linear objective function with an intuitionistic fuzzy coefficients under $DM_t, t = 1, \ldots, T$.
- $\tilde{z}_{tj} = \tilde{d}_{tj}^1 \bar{x}_1 + \tilde{d}_{tj}^2 \bar{x}_2 + \cdots + \tilde{d}_{tj}^T \bar{x}_T$, with $\tilde{d}_{tj}^t \in IF(\mathfrak{R}^{1 \times n_t})$.
- $p_t, t = 1, 2, \ldots, T$ is the number of tth-level linear objective functions.
- The decision control vectors are $\bar{x} = (\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_T) \in \mathfrak{R}^n, \bar{x}_t \in \mathfrak{R}^{n_t}$ with $n = \sum_{t=1}^{T} n_t$.
- $\tilde{S}^I \neq \emptyset$ is an intuitionistic fuzzy convex and bounded feasible set for MO-MLP problem (3).
- An intuitionistic fuzzy coefficient matrix $\tilde{A}_t^I \in IFN(\mathfrak{R}^{m \times n_t})$, $\tilde{B}^I \in IFN(\mathfrak{R}^{m \times 1})$, m for number of constrains.
- The MO-MLP problem (3) is a convex programming problem since all objectives are convex functions and $\tilde{S}^I$ is a convex set.
- All parameters in $\tilde{S}^I$ and coefficient of objective functions $\tilde{d}_{tj}$, are expressed as triangular intuitionistic fuzzy numbers (see Definition 2.2.

3.2. Crisp MO-MLP problem formulation

Using the accuracy function the intuitionistic fuzzy MO-MLP problem Eqs. (3) can be converted into equivalent crisp MO-MLP problem Eqs. (4) as follows:

$$\max_{\bar{x}_1} Z_1(\bar{x}) = \max_{\bar{x}_1}(z_{11}(\bar{x}), z_{12}(\bar{x}), \ldots, z_{1p_1}(\bar{x})) \qquad [\text{1st-}Level]$$

for given $\bar{x}_1$; where, $\bar{x}_2, \bar{x}_3, \ldots, \bar{x}_T$ solves

$$\max_{\bar{x}_2} Z_2(\bar{x}) = \max_{\bar{x}_2}(z_{21}(\bar{x}), z_{22}(\bar{x}), \ldots, z_{2p_2}(\bar{x})) \qquad [\text{2nd-}Level]$$

$$\vdots$$

for given $\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_{T-1}$; where, $\bar{x}_T$ solves

$$\max_{\bar{x}_T} Z_T(\bar{x}) = \max_{\bar{x}_T}(z_{T1}(\bar{x}), z_{T2}(\bar{x}), \ldots, z_{Tp_T}(\bar{x})) \qquad [T\text{th-}Level]$$

Subject to:

$$\bar{x} \in S^c = \{\bar{x} \in \mathfrak{R} | S_\lambda(\tilde{A}_1^I)\bar{x}_1 + S_\lambda(\tilde{A}_2^I)\bar{x}_2 + \cdots + S_\lambda(\tilde{A}_T^I)\bar{x}_T (\geq, \leq, or =) S_\lambda(\tilde{B}^I), \bar{x} \geq \bar{0}\} \tag{4}$$

Where, $z_{tj}(\bar{x}) = S_\lambda(\tilde{z}_{tj}(\bar{x})), \forall t = 1, 2, \ldots, T \ \& \ j = 1, 2, \ldots, p_t$.

4. Proposed solution approach

In many real decision-making processes, a degree of indeterminacy may exist about the members of the decision-making set. In a fuzzy set, only membership is used to represent the degree of belonging for the element (non-membership is considered the complement of membership), but in an intuitionistic fuzzy set, there is also non-membership to represent belonging, non-belonging, and neutrality for the element Atanassov (1986) [22]. Thus, the intuitionistic fuzzy set may be a more general tool for representing this uncertainty-based mathematical model.

Payoff table 4.1

tth-level solitary maximization problem (5)	$z_{t1}(\bar{x})$	$z_{t2}(\bar{x})$	…	$z_{tp_t}(\bar{x})$	$\bar{x}_{tj}^B$
$\max_{\bar{x} \in S^c} z_{t1}(\bar{x})$	$z_{t1}(\bar{x}_{t1}^B)$	$z_{t2}(\bar{x}_{t1}^B)$	…	$z_{tp_t}(\bar{x}_{t1}^B)$	$\bar{x}_{t1}^B$
$\max_{\bar{x} \in S^c} z_{t2}(\bar{x})$	$z_{t1}(\bar{x}_{t2}^B)$	$z_{t2}(\bar{x}_{t2}^B)$	…	$z_{tp_t}(\bar{x}_{t2}^B)$	$\bar{x}_{t2}^B$
⋮	⋮	⋮	…	⋮	⋮
$\max_{\bar{x} \in S^c} z_{tp_t}(\bar{x})$	$z_{t1}(\bar{x}_{tp_t}^B)$	$z_{t2}(\bar{x}_{tp_t}^B)$	…	$z_{tp_t}(\bar{x}_{tp_t}^B)$	$\bar{x}_{tp_t}^B$
Upper tolerance	U_{t1}^μ	U_{t2}^μ	…	$U_{tp_t}^\mu$	
Lower tolerance	L_{t1}^μ	L_{t2}^μ	…	$L_{tp_t}^\mu$	

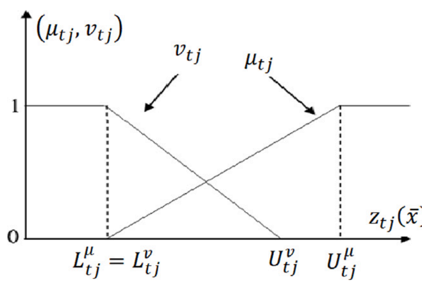

Fig. 1. Membership and non-membership function for each objective function $z_{tj}(\bar{x})$.

4.1. Formulation of membership and non-membership functions

Each decision-maker (DM) in the intuitionistic fuzzy MO-MLP problem is interested in maximizing their intuitionistic fuzzy goal (IFG) at each level over the same constraint set (S^c). These IFGs are presented by the degree of satisfaction (measured by membership) and degree of dissatisfaction (measured by non-membership) of objective functions. To formulate membership and non-membership functions, tolerance and goals should be set first. However, they could hardly be set without meaningful subsidiary data. Therefore, to get the aspiration level and tolerance, we need to find the best optimal value of each objective function at tth-level. That means: the DMs on tth-level individual solve his/her solitary maximization problem as follows:

$$\max_{\bar{x}} z_{tj}(\bar{x}), \ S.t. \quad \bar{x} \in S^c, t = 1, 2, \ldots, T, j = 1, 2, \ldots, p_t. \tag{5}$$

The optimal solution of Eqs. (5), $\bar{x}_{tj}^B = (\bar{x}_1^{tj}, \bar{x}_2^{tj}, \ldots, \bar{x}_T^{tj})$ is considered as the best solution and the corresponding objective value $z_{tj}(\bar{x}_{tj}^B)$ is considered as aspiration level of tjth objective function. Using the individual best solutions, we construct a Payoff table 4.1 for each objective functions in the tth-level.

From a Payoff table 4.1, the maximum and minimum value of each column give upper (U_{tj}^μ) and lower (L_{tj}^μ) tolerance limit, respectively for membership function of tjth objective function ($t = 1, 2, \ldots, T$ indicates tth-level where as, $j = 1, 2, \ldots, p_t$ indicates the objective function in the tth-level). The upper and lower tolerance of non-membership functions are $L_{tj}^\nu = L_{tj}^\mu$ and $U_{tj}^\nu = U_{tj}^\mu - \epsilon_{tj}(U_{tj}^\mu - L_{tj}^\mu)$ respectively, where $\epsilon_{tj} \in (0, 1)$ is predetermined by DM.

The membership and non-membership functions for tjth IFGs can be shown in Fig. 1 and formulated as follows:

$$\mu_{tj}(z_{tj}(\bar{x})) = \begin{cases} 0 & \text{if } z_{tj}(\bar{x}) \le L^{\mu}_{tj} \\ \frac{z_{tj}(\bar{x})-L^{\mu}_{tj}}{U^{\mu}_{tj}-L^{\mu}_{tj}} & \text{if } L^{\mu}_{tj} \le z_{tj}(\bar{x}) \le U^{\mu}_{tj} \\ 1 & \text{if } z_{tj}(\bar{x}) \ge U^{\mu}_{tj} \end{cases}$$
$$\nu_{tj}(z_{tj}(\bar{x})) = \begin{cases} 1 & \text{if } z_{tj}(\bar{x}) \le L^{\nu}_{tj} \\ \frac{U^{\nu}_{tj}-z_{tj}(\bar{x})}{U^{\nu}_{tj}-L^{\nu}_{tj}} & \text{if } L^{\nu}_{tj} \le z_{tj}(\bar{x}) \le U^{\nu}_{tj} \\ 0 & \text{if } z_{tj}(\bar{x}) \ge U^{\nu}_{tj} \end{cases} \quad (6)$$

$t = 1, 2, \ldots, T,\ j = 1, 2, \ldots, p_t$

Using the basic concept of MLP techniques, the IFG of the tth-level decision-maker (DM_t) MOP problem in the intuitionistic fuzzy environment is formulated as follows

$$\text{find} \quad \bar{x}$$
$$\text{Subject to:} \quad z_{tj}(\bar{x}) \gtrsim z_{tj}(\bar{x}^*_{tj}),\ t = 1, 2, \ldots, T,\ j = 1, 2, \ldots, p_t \quad (7)$$
$$\bar{x}_t \cong \bar{x}^*_t,\ t = 1, 2, \ldots, T-1, \quad \bar{x} \in S^c$$

Where, the intuitionistic fuzzy inequality $\gtrsim$ represent the meaning of "substantially greater than or equal to", the intuitionistic fuzzy equality $\cong$ represents the meaning of "approximately" and $\bar{x}^*_t = (\bar{x}^{t*}_1, \bar{x}^{t*}_2, \ldots, \bar{x}^{t*}_T)$, $t = 1, 2, \ldots, T-1$ is Pareto optimal solution to DM_t MOP problem Eqs. (7), must be determined first by using the proposed two-phase IFGP approach. A Pareto optimal solution of DM_t MOP problem Eqs. (7) depend on the optimal solution given by upper level $(\bar{x}^*_l, l = 1, 2, \ldots, t-1)$ and the optimal solution of lower tth– level $t = l+1, l+2, \ldots, T$ obtained individually. That means; $\bar{x}^*_t(\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_{t-1}, \bar{x}^*_{t+1}, \bar{x}^*_{t+2}, \ldots, \bar{x}^*_T)$. The decision control vector $\bar{x}^*_t, t = 1, 2, \ldots, T-1$ for the leader-level are taken as a binding-constraint for lower levels MOP problem. Therefore, the leader level sets range of decision vector $\bar{x}_t, t = 1, 2, \ldots, l-1$ should be "nearly $\bar{x}^*_t$" with positive acceptable relaxation values $\bar{s}^{\mu}_t > 0$ and negative relaxation values $\bar{r}^{\mu}_t > 0$. Those relaxation $\bar{s}^{\mu}_t$ and $\bar{r}^{\mu}_t$ are not necessarily the same.

The degree of optimality for the decision vector controlled by leader-level $t = 1, 2, \ldots, T-1$ is fully acceptable at $\bar{x}^*_t$, strictly increasing over the interval $[\bar{x}^*_t - \bar{r}^{\mu}_t, \bar{x}^*_t]$, strictly decreasing over $[\bar{x}^*_t, \bar{x}^*_t + \bar{s}^{\mu}_t]$, otherwise is zero acceptance by upper-levels. However, due to imprecise decisions the DM may have optimistic approach to word of rejection, zero-acceptance does not mean full rejection. Therefore, the DM has an open-minded view for rejection over the interval $[\bar{x}^*_t - \bar{r}^{\nu}_t, \bar{x}^*_t - \bar{r}^{\mu}_t]$ and $[\bar{x}^*_t + \bar{s}^{\mu}_t, \bar{x}^*_t + \bar{s}^{\nu}_t]$ where, $\bar{r}^{\nu}_t \ge \bar{r}^{\mu}_t > 0$, and/or $0 < \bar{s}^{\mu}_t \le \bar{s}^{\nu}_t$. Since each decision control vector has n_t decision control variable component $\bar{x}^{t*}_t = (x^*_{t1}, x^*_{t2}, \ldots, x^*_{tn_t})$, the positive tolerance $\bar{s}^{\mu}_t = (s^{\mu}_{t1}, s^{\mu}_{t2}, \ldots, s^{\mu}_{tn_t})$, $\bar{s}^{\nu}_t = (s^{\nu}_{t1}, s^{\nu}_{t2}, \ldots, s^{\nu}_{tn_t})$, negative tolerance vector $\bar{r}^{\mu}_t = (r^{\mu}_{t1}, r^{\mu}_{t2}, \ldots, r^{\mu}_{tn_t})$, $\bar{r}^{\nu}_t = (r^{\nu}_{t1}, r^{\nu}_{t2}, \ldots, r^{\nu}_{tn_t})$ and the IFG of decision control variable is $x_{tm} \cong x^*_{tm}$ for $t = 1, 2, \ldots, T-1, m = 1, 2, \ldots, n_t$.

Based on this information, we can formulate the membership, $\mu_{tm}(x_{tm})$ and non-membership functions, $\nu_{tm}(x_{tm})$ as shown in the Fig. 2 to represents the degree of decision control variable for acceptance and rejection by upper tth-level $t = 1, 2, \ldots, T-1$ in the intuitionistic fuzzy environment and defined as follows:

$$\mu_{tm}(x_{tm}) = \begin{cases} \frac{x_{tm}-(x^*_{tm}-r^{\mu}_{tm})}{r^{\mu}_{tm}} & \text{if } x^*_{tm} - r^{\mu}_{tm} \le x_{tm} \le x^*_{tm} \\ \frac{x^*_{tm}+s^{\mu}_{tm}-x_{tm}}{s^{\mu}_{tm}} & \text{if } x^*_{tm} \le x_{tm} \le x^*_{tm} + s^{\mu}_{tm} \\ 0 & \text{otherwise} \end{cases}; \quad (8)$$

$t = 1, 2, \ldots, T-1,\ m = 1, 2, \ldots, n_t$

$$\nu_{tm}(x_{tm}) = \begin{cases} \frac{x^*_{tm}-x_{tm}}{r^{\nu}_{tm}} & \text{if } x^*_{tm} - r^{\nu}_{tm} \le x_{tm} \le x^*_{tm} \\ \frac{x_{tm}-x^*_{tm}}{s^{\nu}_{tm}} & \text{if } x^*_{tm} \le x_{tm} \le x^*_{tm} + s^{\nu}_{tm} \\ 1 & \text{otherwise} \end{cases}; \quad (9)$$

$t = 1, 2, \ldots, T-1,\ m = 1, 2, \ldots, n_t$

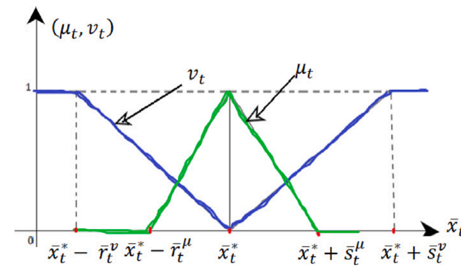

Fig. 2. Membership and non-membership function for decision vector $\bar{x}_t$ controlled by higher-level $t = 1, 2, \ldots, T-1$.

Definition 4.1 ([6,19,21,24]). $\bar{x}^*_t = (\bar{x}^{t*}_1, \bar{x}^{t*}_2, \ldots, \bar{x}^{t*}_T)$ is MN^t-Pareto optimal solution tth-level DM MOP problem, if there does not exist another $\bar{x}_t \in S^c$ such that $\mu_{tj}(z_{tj}(\bar{x}_t)) \ge \mu_{tj}(z_{tj}(\bar{x}^*_t))$, $\&\nu_{tj}(z_{tj}(\bar{x}_t)) \le \nu_{tj}(z_{tj}(\bar{x}^*_t))\forall j = 1, 2, \ldots, p_t$, $\mu_{tm}(x_{tm}) \ge \mu_{tm}(x^*_{tm})\&\nu_{tm}(x_{tm}) \le \nu_{tm}(x^*_{tm}), \forall m$ and strictly inequality is hold for some j for each $t = 1, 2, \ldots, T-1$. Hence, $\bar{x}^* = \{\bar{x}^*_1, \bar{x}^*_2, \ldots, \bar{x}^*_T\}$ is feasible solution of crisp MO-MLP problems Eqs. (4).

Definition 4.2 ([6,21,24]). Let $\bar{x}^* = \{\bar{x}^*_1, \bar{x}^*_2, \ldots, \bar{x}^*_T\}$ is feasible solution of crisp MO-MLP problems Eqs. (4). Then, $\bar{x}^*$ is MN Pareto optimal solution of MO-MLP problems if there does not exist another $\bar{x} \in S^c$ such that $\mu_{1j}(z_{1j}(\bar{x})) \ge \mu_{1j}(z_{1j}(\bar{x}^*)) \wedge \nu_{1j}(z_{1j}(\bar{x})) \le \nu_{1j}(z_{1j}(\bar{x}^*))$, $\forall j = 1, 2, \ldots, p_1$ and $\mu_{1j}(z_{1j}(\bar{x})) > \mu_{1j}(z_{1j}(\bar{x}^*)) \wedge \nu_{1j}(z_{1j}(\bar{x})) < \nu_{1j}(z_{1j}(\bar{x}^*))$, for some $j = 1, 2, \ldots, p_1$.

Definition 4.3 ([6]). Let $\bar{x}^* = \{\bar{x}^*_1, \bar{x}^*_2, \ldots, \bar{x}^*_T\}$ is feasible solution of crisp MO-MLP problems Eqs. (4). Then, $\bar{x}^*$ is Pareto optimal solution of MO-MLP problems if there does not exist another $\bar{x} \in S^c$ such that $z_{1j}(\bar{x}) \ge z_{1j}(\bar{x}^*)$, $\forall j = 1, 2, \ldots, p_1$ and $z_{1j}(\bar{x}) > z_{1j}(\bar{x}^*)$, for some $j = 1, 2, \ldots, p_1$.

Once the IFGs of objective function and decision control variable represented by degree of acceptance $\mu_{tj}(z_{tj}(\bar{x}))$, Eqs. (6-1), $\mu_{tm}(x_{tm})$, Eqs. (8) and rejection $\nu_{tj}(z_{tj}(\bar{x}))$, Eqs. (6-2), $\nu_{tm}(x_{tm})$ Eqs. (9). Also known that the highest value of membership $\mu_{tj}(z_{tj}(\bar{x})), \mu_{tm}(x_{tm})$ is one (1), non-membership $\nu_{tj}(z_{tj}(\bar{x})), \nu_{tm}(x_{tm})$ is zero (0) for IFGs. Following this, we formulate the standard goal programming problem in the intuitionistic fuzzy environment for both upper tth-level MOP problems (7) and crisp MO-MLP problems (4).

4.2. An IFGP model for MO-MLP problems

In an intuitionistic fuzzy environment, the plan of each decision-maker (DM) is to attain the highest degree of achievement. Therefore, the classical goal programming approach is used to minimize the unwanted deviation variables of the DM from all levels. In the proposed IFGP approach, the IFGs are characterized by the linear membership and non-membership functions for tjth objective functions, Eqs. (6) and decision control variable, x_{tm}, Eqs. (8), Eqs. (9) are transformed into flexible goals with unity (1) and zero (0) respectively by means of, under-deviation $D^{-\mu}_{tj}, d^{-\mu}_{tm}, D^{-\nu}_{tj}, d^{-\nu}_{tm}$ and over-deviation $D^{+\mu}_{tj}, d^{+\mu}_{tm}, D^{+\nu}_{tj}, d^{+\nu}_{tm}$ variables from aspiration levels. Thus the converted intuitionistic fuzzy goal programming equations can be signified as follows:

$$\begin{aligned} \mu_{tj}(z_{tj}(\bar{x})) + D^{-\mu}_{tj} - D^{+\mu}_{tj} &= 1, \forall t = 1, 2, \ldots, T; j = 1, 2, \ldots, p_t \\ \nu_{tj}(z_{tj}(\bar{x})) + D^{-\nu}_{tj} - D^{+\nu}_{tj} &= 0, \forall t = 1, 2, \ldots, T; j = 1, 2, \ldots, p_t \\ \mu_{tm}(x_{tm}) + d^{-\mu}_{tm} - d^{+\mu}_{tm} &= 1, \forall t = 1, 2, \ldots, T-1, m = 1, 2, \ldots, n_t \\ \nu_{tm}(x_{tm}) + d^{-\nu}_{tm} - d^{+\nu}_{tm} &= 0, \forall t = 1, 2, \ldots, T-1, m = 1, 2, \ldots, n_t \end{aligned} \quad (10)$$

Since the over-attainment, $D^{+\mu}_{tj}, d^{+\mu}_{tm}$ and under-attainment, $D^{-\nu}_{tj}, d^{-\nu}_{tm}$ from an IFGs are giving highest achievement and lowest achievement of membership and non-membership functions respectively Rukmani and Porchelvi (2018) [20], then those variables have value zeros in the Eqs. (10). Therefore, the above Eqs. (10) is reduced to:

$$\mu_{tj}(z_{tj}(\bar{x})) + D_{tj}^{-\mu} \geq 1; \nu_{tj}(z_{tj}(\bar{x})) - D_{tj}^{+\nu} \leq 0,\ \forall t = 1,2,\ldots,T; j = 1,2,\ldots,p_t$$
$$\mu_{tm}(x_{tm}) + d_{tm}^{-\mu} \geq 1;\ \nu_{tm}(x_{tm}) - d_{tm}^{+\nu} \leq 0 \forall t = 1,2,\ldots,T-1, m = 1,2,\ldots,n_t \tag{11}$$

Now using the goal programming problem in the intuitionistic fuzzy environment Eqs (11) at the same significant level, the proposed **Phase-I** IFGP Model for Multi-Objective Multilevel Programming (MO-MLP) problems can be modeled as follows:

Phase-I IFGP Model:

$$\min \sum_{t=1}^{T}\sum_{j=1}^{p_t}(W_{tj}^{-\mu}D_{tj}^{-\mu} + W_{tj}^{+\nu}D_{tj}^{+\nu})$$
$$+\sum_{t=1}^{T-1}\sum_{m=1}^{n_t}[w_{tm}^{1\mu}d_{tm}^{-1\mu} + w_{tm}^{2\mu}d_{tm}^{-2\mu} + w_{tm}^{1\nu}d_{tm}^{+1\nu} + w_{tm}^{2\nu}d_{tm}^{+2\nu}]$$
Subject to:
$$\mu_{tj}(z_{tj}(\bar{x})) + D_{tj}^{-\mu} \geq 1;\ \nu_{tj}(z_{tj}(\bar{x})) - D_{tj}^{+\nu} \leq 0, \forall t = 1,2,\ldots,T;\quad j = 1,2,\ldots,p_t \tag{12}$$
$$\mu_{tm}^{1}(x_{tm}) + d_{tm}^{-1\mu} \geq 1;\ \nu_{tm}^{1}(x_{tm}) - d_{tm}^{+1\nu} \leq 0, \forall t = 1,2,\ldots,T-1;\quad m = 1,2,\ldots,n_t$$
$$\mu_{tm}^{2}(x_{tm}) + d_{tm}^{-2\mu} \geq 1;\ \nu_{tm}^{2}(x_{tm}) - d_{tm}^{+2\nu} \leq 0, \forall t = 1,2,\ldots,T-1;\quad m = 1,2,\ldots,n_t$$
$$\bar{x} \in S^c;\quad D_{tj}^{-\mu}, D_{tj}^{+\nu}, d_{tm}^{-1\mu}, d_{tm}^{+1\nu}, d_{tm}^{-2\mu}, d_{tm}^{+2\nu} \geq 0$$

Where, from Eqs. (8) and (9) we have $\mu_{tm}(x_{tm}) = (\mu_{tm}^{1}(x_{tm}), \mu_{tm}^{2}(x_{tm}))$, $\nu_{tm}(x_{tm}) = (\nu_{tm}^{1}(x_{tm}), \nu_{tm}^{2}(x_{tm}))$, then the corresponding deviations in Eqs. (12) are $d_{tm}^{-\mu} = (d_{tm}^{-1\mu}, d_{tm}^{-2\mu})$ and $d_{tm}^{+\nu} = (d_{tm}^{+2\nu}, d_{tm}^{+2\nu})$, the numerical coefficient $W_{tj}^{-\mu}, W_{tj}^{+\nu}$, $w_{tm}^{1\mu}, w_{tm}^{2\mu}, w_{tm}^{1\nu}$ and $w_{tm}^{2\nu}$ represents the relative importance relation of achieving the target levels of the respective IFGs in the decision situations. To measure the relative importance of IFGs appropriately, the weighting scheme proposed by Mohamed (1997) [17] can be used to give the values to $W_{tj}^{-\mu}, W_{tj}^{+\nu}$, $w_{tm}^{1\mu}, w_{tm}^{2\mu}, w_{tm}^{1\nu}$, and $w_{tm}^{2\nu}$. These values are calculated as:

$$W_{tj}^{-\mu} = \frac{1}{U_{tj}^{\mu} - L_{tj}^{\mu}}, \text{ and } W_{tj}^{+\nu} = \frac{1}{U_{tj}^{\nu} - L_{tj}^{\nu}} \tag{13}$$
$$w_{tm}^{1\mu} = \frac{1}{r_{tm}^{-\mu}};\ w_{tm}^{2\mu} = \frac{1}{s_{tm}^{+\mu}};\ w_{tm}^{1\nu} = \frac{1}{r_{tm}^{-\nu}} \text{ and } w_{tm}^{2\nu} = \frac{1}{s_{tm}^{+\nu}} \tag{14}$$

Theorem 4.1. *A unique MN-Pareto optimal solution of proposed* ***Phase-I*** *IFGP Model* (12) *is also a Pareto optimal solution to the crisp MO-MLP problem* (4).

Proof. Suppose $\bar{x}^*$ is unique MN-Pareto optimal solution to problem (12). Then $\bar{x}^*$ is Pareto optimal solution to tth-level MOP problem Singh and Yadav (2015) [19], El Sayed and Abo-Sinna (2021) [27] and $\mu_{1j}(z_{1j}(\bar{x}^*)) > \mu_{1j}(z_{1j}(\bar{x})) \wedge \nu_{1j}(z_{1j}(\bar{x}^*)) < \nu_{1j}(z_{1j}(\bar{x}))$ for all $j = 1,2,\ldots,p_1$. Assume that $\bar{x}^*$ is not Pareto optimal solution to crisp MO-MLP problem. Then there exist $\bar{x} \in S^c$ such that $z_{1j}(\bar{x}^*) \leq z_{1j}(\bar{x})$ $\forall j = 1,2,\ldots,p_1$ and $z_{1j}(\bar{x}^*) < z_{1j}(\bar{x})$ for some j. Then we have
$z_{1j}(\bar{x}^*) - L_{1j}^{\mu} \leq z_{1j}(\bar{x}) - L_{1j}^{\mu}, \forall j$ and $z_{1j}(\bar{x}^*) - L_{1j}^{\mu} < z_{1j}(\bar{x}) - L_{1j}^{\mu}$, for some j.
$U_{1j}^{\nu} - z_{1j}(\bar{x}^*) \geq U_{1j}^{\nu} - z_{1j}(\bar{x}), \forall j$ and $U_{1j}^{\nu} - z_{1j}(\bar{x}^*) > U_{1j}^{\nu} - z_{1j}(\bar{x})$, for some j.
Since $U_{1j}^{\mu} - L_{1j}^{\mu}, U_{1j}^{\nu} - L_{1j}^{\nu} > 0$, we get
$\frac{z_{1j}(\bar{x}^*)-L_{1j}^{\mu}}{U_{1j}^{\mu}-L_{1j}^{\mu}} \leq \frac{z_{1j}(\bar{x})-L_{1j}^{\mu}}{U_{1j}^{\mu}-L_{1j}^{\mu}}, \forall j$ and $\frac{z_{1j}(\bar{x}^*)-L_{1j}^{\mu}}{U_{1j}^{\mu}-L_{1j}^{\mu}} > \frac{z_{1j}(\bar{x})-L_{1j}^{\mu}}{U_{1j}^{\mu}-L_{1j}^{\mu}}$, for some j.
$\frac{U_{1j}^{\nu}-z_{1j}(\bar{x}^*)}{U_{1j}^{\nu}-L_{1j}^{\nu}} \geq \frac{U_{1j}^{\nu}-z_{1j}(\bar{x}^*)}{U_{1j}^{\nu}-L_{1j}^{\nu}}, \forall j$ and $\frac{U_{1j}^{\nu}-z_{1j}(\bar{x}^*)}{U_{1j}^{\nu}-L_{1j}^{\nu}} > \frac{U_{1j}^{\nu}-z_{1j}(\bar{x}^*)}{U_{1j}^{\nu}-L_{1j}^{\nu}}$, for some j. This implies that $\mu_{1j}(z_{1j}(\bar{x}^*)) \leq \mu_{1j}(z_{1j}(\bar{x})), \forall j$ and $\mu_{1j}(z_{1j}(\bar{x}^*)) < \mu_{1j}(z_{1j}(\bar{x}))$, for some j. and similarly we get $\nu_{1j}(z_{1j}(\bar{x}^*)) \geq \nu_{1j}(z_{1j}(\bar{x})), \forall j$ and $\nu_{1j}(z_{1j}(\bar{x}^*)) > \nu_{1j}(z_{1j}(\bar{x}))$, for some j. This show that $\bar{x}^*$ is also MN-Pareto optimal solution to problem (12). Hence, we arrived at contradiction to $\bar{x}^*$ is unique MN-Pareto optimal solution to problem (12). Therefore, $\bar{x}^*$ is Pareto optimal solution to crisp MO-MLP problem (4)

The main challenge occur in the literature of optimizetion method which used to solve MO-MLP problem under imprecise environment generated an MN-Pareto optimal solution, this solution was considered as Pareto optimal solution of MO-MLP problems [1,5–9,18]. "But the case $\bar{x}^* \in S^c$ is MN-Pareto optimal solution to MO-MLP problem and $\mu_{1j}(z_{1j}(\bar{x}^*)) = 1$ for some $j = 1,2,\ldots,p_1$ may not be Pareto optimal solution to MO-MLP problem" Jimenez and Bibao (2009) [30]. This is due to the fact that $\mu_{1j}(z_{1j}(\bar{x}^*)) = 1$ for any $z_{1j}(\bar{x}^*) \geq U_{1j}$ for instance if $\bar{y}^* \in S^c$ such that for $j = 1,2,\ldots,p_1$, $z_{1j}(\bar{y}^*) > z_{1j}(\bar{x}^*) \geq U_{1j}$, then $\bar{x}^*$ is MN-Pareto optimal solution but not Pareto optimal solution to MO-MLP problem. Several authors Jimenez and Bibao (2009) [30], Chen L. and Chen H. (2015) [31], Razmi et al. (2016) [21] have proposed different techniques to obtain Pareto optimal solution to MOP problem.

In this paper, we extended the approach proposed by Jimenez and Bibao (2009) [30] in the fuzzy environment into an intuitionistic fuzzy environment, to overcome the challenge of obtaining Pareto optimal solution. Therefore, we proposed the **phase-II IFGP** model to solve DM_t MOP problem. Let $\bar{x}^{**}$ is MN-Pareto optimal solution to **phase-I IFGP model** (12) for DM_t MOP problem, $F = \{j = 1,2,\ldots,p_t | \mu_{tj}(z_{tj}(\bar{x}^{**})) = 1 \text{ for each } t = 1,2,\ldots T\}$, $G = \{j = 1,2,\ldots,p_t | \mu_{tj}(z_{tj}(\bar{x}^{**})) < 1 \text{ for each } t = 1,2,\ldots T\}$ and $z_{ti}(\bar{x}^{**}) \neq 0$, otherwise the DM used approximate value to it.

$$W_{ti} = \frac{1}{|(z_{ti}(\bar{x}^{**}))(U_{ti}^{\mu} - L_{ti}^{\mu})|},\ i \in F \tag{15}$$

The Phase-II IFGP model for DM_t MOP problem defined as follows:

Phase-II IFGP Model:

$$\max \sum_{i\in F}^{p_t} W_{ti}D_{ti} \qquad t = 1,2,\ldots,T$$
Subject to:
$$z_{ti}(\bar{x}) - D_{ti} = z_{ti}(\bar{x}^{**});\ \forall i \in F, \text{ for each } t = 1,2,\ldots,T$$
$$\mu_{tl}(z_{tl}(\bar{x})) = \mu_{tl}(z_{tl}(\bar{x}^{**}));\ \nu_{tl}(z_{tl}(\bar{x})) = \nu_{tl}(z_{tl}(\bar{x}^{**}))\,\forall l \in G \quad \text{for each } t = 1,2,\ldots,T \tag{16}$$
$$\mu_{tm}(x_{tm}) = \mu_{tm}(x_{tm}^{**});\ \nu_{tm}(x_{tm}) = \nu_{tm}(x_{tm}^{**}), \forall t = 1,2,\ldots,T-1;\quad m = 1,2,\ldots,n_t$$
$$\bar{x} \in S^c;\quad D_{ti} \geq 0,\ \forall i \in F$$

Where, the weight W_{ti}, $i \in F$ is pre-calculated using Eqs. (15), D_{ti}, $i \in F$ represent the positive deviation from $z_{ti}(\bar{x}^{**})$ for each $t = 1,2,\ldots,T$.

Theorem 4.2. *Let an optimal solution of model* (16) *be* $\hat{x}$, *all objective functions are continuous and* S^I *is compact set in the convex programming, MO-MLP problem* (3). *Then the solution* $\hat{x}$ *satisfy both MN-Pareto optimal to intuitionistic fuzzy MO-MLP problem* (3) *and Pareto optimal to crisp MO-MLP problem* (4).

Proof. Similar to Theorem 4.1 and in the literature recorded by Jimenez and Bibao (2009) [30].

4.3. Proposed solution algorithm for MO-MLP problem

Step-1 Using accuracy function (S_λ, $\lambda = \frac{1}{2}$), convert the intuitionistic fuzzy MO-MLP problem Eqs. (3) into crisp MO-MLP problem Eqs. (4).

Step-2 Using a single objective function for a time, model the solitary maximization problem under a crisp feasible set for each objective function, Eqs. (5) and solve it. The objective value at each best/ideal-solution obtained from Eqs. (5) is considered to be or determined goal for each objective function.

Step-3 Using the objective value at best/ideal-solutions construct a payoff table as discussed above Table 4.1 to determine upper (goal) and lower tolerance limits for all objective functions in all levels.

Step-4 Set $t = 1$ to formulate tth-level MOP problems as shown in Eqs. (7)

Step-5 Calculate the weight of IFGs of objective functions present in Eqs. (13).

Step-6 Build the linear membership, $\mu_{tj}(z_{tj}(\bar{x}))$ and non-membership function, $\nu_{tj}(z_{tj}(\bar{x}))$ for each objective function $z_{tj}(\bar{x}), j = 1, 2, \dots, p_t$. See Eqs. (6).

Step-7 Solve **Phase-I IFGP** model (12) at tth−level and then we get the MN^t−Pareto optimal solution, $\bar{x}_t^* = (\bar{x}_1^{t*}, \bar{x}_2^{t*}, \dots, \bar{x}_T^{t*})$.

Step-8 If $\mu_{tj}(z_{tj}(\bar{x}_t^*)) < 1, \forall j = 1, 2, \dots, p_t$, $\bar{x}_t^*$ is Pareto optimal solution to tth-level MOP problems. Then go to **Step-9**. Otherwise, (i.e. $\mu_{tj}(z_{tj}(\bar{x}_t^*)) = 1$ for some j) solve **Phase-II IFGP** model (16), then go to **Step-9**.

Step-9 Set the maximum acceptance (membership) and rejection (non-membership) range for each decision control variable $\bar{x}_{tm}^*, m = 1, 2, \dots, n_t$ as positive tolerance limit is $\bar{s}_{tm}^{\mu}, \bar{s}_{tm}^{\nu}$ and negative tolerance limit is $r_{tm}^{\mu}, r_{tm}^{\nu}$.

Step-10 Build the membership, $\mu_{tm}(x_{tm})$ and non-membership function $\nu_{tm}(x_{tm}), m = 1, 2, \dots, n_t$ of decision control variable $\bar{x}_t = (x_{t1}, x_{t2}, \dots, x_{tn_t})$ as shown in Eqs. (8) and (9).

Step-11 Calculate the weight of IFG of decision control variable as shown in Eqs. (14).

Step-12 If $t = T$, T is the total number of levels, go to **Step-13**. Otherwise, $t = t + 1$ and go to **Step-5**

Step-13 Formulate the final **Phase-I IFGP** model of MO-MLP problem as Eqs. (12) and solve it, to get a solution $\bar{x}_T^* = (\bar{x}_1^{T*}, \bar{x}_2^{T*}, \dots, \bar{x}_T^{T*})$. If $\mu_{Tj}(z_{Tj}(\bar{x}_T^*)) < 1, \forall j = 1, 2, \dots, p_t$, go to **Step-14**. Otherwise, solve **Phase-II IFGP** model (16) then go to **Step-14**.

Step-14 If all decision-makers are satisfied with the solution in **Step-13**, stop the Pareto optimal solution is reached. Otherwise, improve the tolerance of all objective function and/ or decision control variable, then go to **Step-4**.

Definition 4.4 (*Distance Measure Function*)**.** To compare the performances of different methods available in the literature for solving various domains of MO-MLP (3) problem, we developed the metric distance measure function, $d(\bar{x})$ as follows:

$$d(\bar{x}) = \frac{\sqrt{\sum_{t=1}^{T} \sum_{j=1}^{p_t} (z_{tj}(\bar{x}^B) - z_{tj}(\bar{x}))^2}}{2 \times \sum_{t=1}^{T} p_t} \tag{17}$$

Where p_t is the number of linear objective function under tth-level DM in the MO-MLP (3) problem, $z_{tj}(\bar{x}^B)$ is aspiration level (goal) set by DMs or Ideal value of objective function $z_{tj}(\bar{x})$, $\bar{x} \in S$ i.e., $z_{tj}(\bar{x}^B) = \max_{\bar{x} \in S} z_{tj}(\bar{x})$. In this paper we used the Ideal values. The smaller value of $d(\bar{x})$ is indicated that $\bar{x}$ is the preferable compromise solution, [5].

5. Numerical illustration

To demonstrate and evaluate the efficiency of the proposed two-phase IFGP method, we considered two different numerical examples.

5.1. Example 1: (Ref. Baky (2010) [1], Arbaiy & Wetade (2012) [8])

Let us use the first numerical example exactly from Baky's (2010) [1] and Arbaiy & Wetade's (2012) [8] papers. Hint: $\max_{\bar{x} \in S} -f_{tj}(\bar{x}) \equiv \min_{\bar{x} \in S} f_{tj}(\bar{x})$.

Table 5.1
The numerical values U_{tj}^{μ}, U_{tj}^{ν}, $L_{tj}^{\mu} = L_{tj}^{\nu}$, W_{tj}^{μ} and W_{tj}^{ν} for each $f_{tj}(\bar{x})$.

	$f_{11}(\bar{x})$	$f_{12}(\bar{x})$	$f_{21}(\bar{x})$	$f_{22}(\bar{x})$	$f_{23}(\bar{x})$	$f_{31}(\bar{x})$	$f_{32}(\bar{x})$
$U_{tj}^{\mu} = \max_{\bar{x} \in S} f_{tj}(\bar{x})$	2.5	3.5	1	1	1	0.5	0
$L_{tj}^{\mu} = L_{tj}^{\nu} = \min_{\bar{x} \in S} f_{tj}(\bar{x})$	−1	−3	−4	−2	−5	−8.5	−2
U_{tj}^{ν}	2.495	3.495	0.595	0.595	0.595	0.495	−0.005
W_{tj}^{μ}	0.286	0.154	0.2	0.33	0.167	0.111	0.5
W_{tj}^{ν}	0.2861	0.1541	0.22	0.385	0.178	0.1112	0.51

$$\max_{x_1} (f_{11}(\bar{x}) = -x_1 + x_2 + 4x_3; f_{12}(\bar{x}) = x_1 - 3x_2 + 4x_3) \quad \text{[1st-}Level\text{]}$$

given x_1, where x_2 and x_3 solved by

$$\max_{x_2} (f_{21}(\bar{x}) = -2x_1 + x_2 - 2x_3; f_{22}(\bar{x}) = -2x_1 - x_2 + 3x_3; f_{23}(\bar{x}) = -3x_1 + x_2 - x_3) \, \text{[2nd-}Level\text{]}$$

given x_1 and x_2 where x_3 solved by

$$\max_{x_3} (f_{31}(\bar{x}) = -7x_1 - 3x_2 + 4x_3; f_{32}(\bar{x}) = -x_1 - x_3) \quad \text{[3rd-}Level\text{]}$$

Subject to:

$$S = \left\{ \begin{array}{lll} x_1 + x_2 + x_3 \leq 3, & x_1 + x_2 - x_3 \leq 1, & x_1 + x_2 + x_3, \geq 1 \\ -x_1 + x_2 + x_3 \leq 1, & x_3 \leq 0.5, & x_1, x_2, x_3 \geq 0, \end{array} \right\} \tag{18}$$

Solution. Step 1: All parameters of Example 1 (5.1) are given as crisp numbers

Step 2: Solve $\max_{\bar{x} \in S} f_{tj}(\bar{x})$ and $\min_{\bar{x} \in S} f_{tj}(\bar{x})$ individually for each objective function $f_{tj}(\bar{x})$.

Step 3: The decision-makers set the upper tolerance $U_{tj}^{\mu} = \max_{\bar{x} \in S} f_{tj}(\bar{x})$, $U_{tj}^{\nu} = U_{tj}^{\mu} - \epsilon_{tj} \, (U_{tj}^{\mu} - L_{tj}^{\mu})$, $\epsilon_{tj} \in (0, 1)$ and the lower tolerance limit $L_{tj}^{\mu} = L_{tj}^{\nu} = \min_{\bar{x} \in S} f_{tj}(\bar{x})$ as shown in Table 5.1.

Step 4 & 5: Using Eqs. (13) calculate the weight W_{tj}^{μ} and W_{tj}^{ν} of IFG for each $f_{tj}(\bar{x})$, shown in Table 5.1.

Step 6 & 7: Solve **Phase-I IFGP model for DM_1 MOP problem of Example 1** (5.1)**:** see the equation in Box I.

Using LINGO-19 software solve the above model we obtained optimal solution $\bar{x}_1^* = (x_{11}^*, x_{12}^*, x_{13}^*)$, where, $x_{11}^* = 0.0025, x_{12}^* = 0.4975, x_{13}^* = 0.5$.

Step 8: $\mu_{11}(f_{11}(\bar{x}_1^*)), \mu_{12}(f_{12}(\bar{x}_1^*)) < 1$.

Step 9, 10 & 11: First-level decision-maker set positive and negative tolerance for decision control variable $x_{11}^* = 0.0025$ as $r_{11}^{\mu} = s_{11}^{\mu} = 0.5$ and $r_{11}^{\nu} = s_{11}^{\nu} = 1$. The weight of decision variable x_1 controlled by leader DM using Eqs. (14) is $w_{11}^{-1\mu} = w_{11}^{-2\mu} = 2$ and $w_{11}^{+1\nu} = w_{11}^{+2\nu} = 1$.

Step 12: Since $t < 3$, go to step 5.

Step 5, 6, & 7: Solve **Phase-I IFGP model for DM_2 MOP problem of Example 1** (5.1)**:** see the equation in Box II.

Solve the above single objective programming model using LINGO-19 software, we obtained an optimal solutions $\bar{x}_2^* = (x_{21}^*, x_{22}^*, x_{23}^*)$, where, $x_{21}^* = 0.0025, x_{22}^* = 0.5025, x_{23}^* = 0.4998$.

Step 8: $\mu_{21}(f_{21}(\bar{x}_2^*)), \mu_{22}(f_{22}(\bar{x}_2^*)) \& \mu_{23}(f_{23}(\bar{x}_2^*)) < 1$.

Step 9, 10 & 11: The second-level decision-maker decides positive and negative tolerance limit for decision control variable $x_{22}^* = 0.5025$ as $r_2^{\mu} = 0.75, s_2^{\mu} = 0.25$ and $r_2^{\nu} = 1, s_2^{\nu} = 0.5$. The weight of decision variable x_{22} controlled by DM_2 using Eqs. (14) is $w_{22}^{-1\mu} = 1.33, w_{22}^{-2\mu} = 4$, $w_{22}^{+1\nu} = 1, \& w_{22}^{+2\nu} = 2$.

Step 12: Since $t = 3$, go to step 13:

Step 13: Solve **Phase-I IFGP model for DM_3 MOP problem of Example 1** (5.1)**:** see the equation in Box III.

Final Step 14: Solve the above linear programming problem using LINDO-19, then the decision-makers considered its solution $\bar{x}^* = (0.0025, 0.5025, 0.5)$ as Pareto-optimal solution to the MO-MLP problem of a numerical example 1 (5.1):

$$\min 0.286D_{11}^{-\mu} + 0.2861D_{11}^{+\nu} + 0.154D_{12}^{-\mu} + 0.1541D_{12}^{+\nu}$$
$$\text{Subject to: } \left\{ \begin{array}{ll} -x_1 + x_2 + 4x_3 + 3.5D_{11}^{-\mu} \geq 2.5, & x_1 - x_2 - 4x_3 - 3.495D_{11}^{+\nu} \leq -2.495, \\ x_1 - 3x_2 + 4x_3 + 6.5D_{12}^{-\mu} \geq 3.5, & -x_1 + 3x_2 - 4x_3 - 6.495D_{12}^{+\nu} \leq -3.495, \\ \bar{x} = (x_1, x_2, x_3) \in S & Eqs. (18) \end{array} \right\} \tag{19}$$

Box I.

$$min\, 0.286D_{11}^{-\mu} + 0.2861D_{11}^{+\nu} + 0.154D_{12}^{-\mu} + 0.1541D_{12}^{+\nu} + 0.2D_{21}^{-\mu} + 0.22D_{21}^{+\nu} + 0.33D_{22}^{-\mu} + 0.385D_{22}^{+\nu}$$
$$+ 0.167D_{23}^{-\mu} + 0.178D_{23}^{+\nu} + 2d_1^{-1\mu} + 2d_1^{-2\mu} + d_1^{+1\nu} + d_1^{+2\nu}$$
$$\text{Subject to: } \left\{ \begin{array}{ll} -2x_1 + x_2 - 2x_3 + 5D_{21}^{-\mu} \geq 1, & 2x_1 - x_2 + 2x_3 - 4.595D_{21}^{+\nu} \leq -0.595, \\ -2x_1 - x_2 + 3x_3 + 3D_{22}^{-\mu} \geq 1, & 2x_1 + x_2 - 3x_3 - 2.595D_{22}^{+\nu} \leq -0.595, \\ -3x_1 + x_2 - x_3 + 6D_{23}^{-\mu} \geq 1, & 3x_1 - x_2 + x_3 - 2.595D_{23}^{+\nu} \leq -0.595, \\ x_1 + 0.5d_1^{-1\mu} \geq 0.0025, & -x_1 + 0.5d_1^{-2\mu} \geq -0.0025, \\ -x_1 - d_1^{+1\nu} \leq -0.0025, & x_1 - d_1^{+2\nu} \leq 0.0025, \\ \text{With the system constraint} & (19) \end{array} \right\} \tag{20}$$

Box II.

$$min\, 0.286D_{11}^{-\mu} + 0.2861D_{11}^{+\nu} + 0.154D_{12}^{-\mu} + 0.1541D_{12}^{+\nu} + 0.2D_{21}^{-\mu} + 0.22D_{21}^{+\nu} + 0.33D_{22}^{-\mu} + 0.385D_{22}^{+\nu}$$
$$+ 0.167D_{23}^{-\mu} + 0.178D_{23}^{+\nu} + 2d_1^{-1\mu} + 2d_1^{-2\mu} + d_1^{+1\nu} + d_1^{+2\nu} + 0.111D_{31}^{-\mu} + 0.1112D_{31}^{+\nu} + 0.5D_{32}^{-\mu} + 0.52D_{32}^{+\nu}$$
$$+ 1.33d_2^{-1\mu} + 4d_2^{-2\mu} + d_2^{+1\nu} + 2d_2^{+2\nu}$$
$$\text{Subject to: } \left\{ \begin{array}{ll} -7x_1 - 3x_2 + 4x_3 + 9D_{31}^{-\mu} \geq 0.5, & 7x_1 + 3x_2 - 4x_3 - 9.495D_{31}^{+\nu} \leq -0.495 \\ -x_1 - x_3 + 2D_{32}^{-\mu} \geq 0, & x_1 + x_3 - 1.995D_{32}^{+\nu} \leq 0.005 \\ x_2 + 0.75d_2^{-1\mu} \geq 0.5025, & -x_2 + 0.25d_2^{-2\mu} \geq -0.5025 \\ -x_2 - d_2^{+1\nu} \leq -0.5025, & x_2 - 0.5d_2^{+2\nu} \leq 0.5025, \\ \text{With the system constraint} & (20) \end{array} \right\} \tag{21}$$

Box III.

Table 5.1.1
Comparison of Pareto-optimal solution for numerical example 1 (5.1): based on the proposed Two-phase IFGP method, the FGP approach of Baky (2010) [1] and the weighted additive FGP method of Arbaiy and Watada (2012) [8].

$\max_{\bar{x}\in S} f_{tj}(\bar{x})$	Proposed method	Baky approach [1]	Arbaiy and Watada approach [8]
$f_{11}(\bar{x}) = 2.5$	$f_{11}(\bar{x}) = 2.5$	$f_{11}(\bar{x}) = 2.498$	$f_{11}(\bar{x}) = 2.487$
$f_{12}(\bar{x}) = 3.5$	$f_{12}(\bar{x}) = 0.495$	$f_{12}(\bar{x}) = -0.494$	$f_{12}(\bar{x}) = 0.472$
$f_{21}(\bar{x}) = 1$	$f_{21}(\bar{x}) = -0.502$	$f_{21}(\bar{x}) = -1.002$	$f_{21}(\bar{x}) = -0.489$
$f_{22}(\bar{x}) = 1$	$f_{22}(\bar{x}) = 0.992$	$f_{22}(\bar{x}) = -0.498$	$f_{22}(\bar{x}) = 0.983$
$f_{23}(\bar{x}) = 1$	$f_{23}(\bar{x}) = -0.005$	$f_{23}(\bar{x}) = -1.002$	$f_{23}(\bar{x}) = 0.006$
$f_{31}(\bar{x}) = 0.5$	$f_{31}(\bar{x}) = 0.475$	$f_{31}(\bar{x}) = -4.493$	$f_{31}(\bar{x}) = 0.468$
$f_{32}(\bar{x}) = 0$	$f_{32}(\bar{x}) = -0.502$	$f_{32}(\bar{x}) = -1$	$f_{32}(\bar{x}) = -0.496$
$\bar{x}$	$\bar{x}^* = (0.0025, 0.5025, 0.5)$	$\hat{x} = (0.5, 0.998, 0.5)$	$\tilde{x} = (0.0005, 0.504, 0.496)$
$d(\bar{x}) = 0$	$d(\bar{x}^*) = 0.253030$	$d(\hat{x}) = 0.515782$	$d(\tilde{x}) = 0.253762$

5.1.1. Comparison with FGP method for Example 1 (5.1)

The solution for the MO-MLP problem of a numerical example 1 (5.1): results by Baky (2010) [1] FGP approach is $\hat{x} = (0.5, 0.998, 0.5)$ and results by Arbaiy and Watada (2012) [8] weighted-additive FGP approach is $\tilde{x} = (0.0005, 0.504, 0.496)$. Table 5.1.1 presents the comparison between the solution $\bar{x}^* = (0.0025, 0.5025, 0.5)$ obtained by proposed method, $\hat{x} = (0.5, 0.998, 0.5)$ given by Baky (2010) [1] and $\tilde{x} = (0.0005, 0.504, 0.496)$ generated by Arbaiy and Watada (2012) [8] for the same numerical example 1 (5.1). As shown in the Table 5.1.1 results, the value of distance measure $d(\bar{x})$ calculated using Eqs. (17) is obtained:

$$d(\bar{x}^*) = 0.253030, d(\hat{x}) = 0.515782, d(\tilde{x}) = 0.253762. \tag{22}$$

The less value of $d(\bar{x})$ show that the higher satisfaction level for Pareto-optimal solution $\bar{x}$ of decision-makers. This results indicate that our optimal solution $\bar{x}^*$ dominates both the optimal solution $\hat{x}$ given by Baky (2010) [1] and $\tilde{x}$ given by Arbaiy and Watada (2012) [8], due to the value of $d(\bar{x})$ as:

$$d(\bar{x}^*) < d(\bar{x}) < d(\hat{x}) \tag{23}$$

5.2. Example 2

In this subsection, we considered a MO-MLP problem with all coefficients of the objective function, coefficient of constraint function and the resource quantity being triangular intuitionistic fuzzy numbers.

$$\max_{x_1}(\tilde{z}^I_{11}(\bar{x}) = \tilde{7}^I x_1 + \tilde{3}^I x_2 - \tilde{4}^I x_3; \tilde{z}^I_{12}(\bar{x}) = \tilde{1}^I x_1 + \tilde{3}^I x_2 + \tilde{4}^I x_3) \quad \text{[1st-}Level\text{]}$$

given x_1, where x_2 and x_3 solves

$$\max_{x_2}(\tilde{z}^I_{21}(\bar{x}) = \tilde{5}^I x_1 - \tilde{2}^I x_2 + \tilde{1}^I x_3; \tilde{z}^I_{22}(\bar{x}) = \tilde{1}^I x_1 + \tilde{1}^I x_2 + \tilde{1}^I x_3; \tilde{z}^I_{23}(\bar{x}) = -\tilde{2}^I x_1 + \tilde{1}^I x_2 + \tilde{2}^I x_3) \ \text{[2nd-}Level\text{]}$$

given x_1, x_2, where x_3 solve

$$\max_{x_3}(\tilde{z}^I_{31}(\bar{x}) = \tilde{3}^I x_1 - \tilde{2}^I x_2 + \tilde{2}^I x_3; \tilde{z}^I_{32}(\bar{x}) = \tilde{5}^I x_2 + \tilde{4}^I x_3) \quad \text{[3rd-}Level\text{]}$$

Subject to:

$$S^I = \left\{ \begin{array}{ll} \tilde{1}^I x_1 + \tilde{1}^I x_2 + \tilde{1}^I x_3 \le \tilde{5}^I & \tilde{1}^I x_1 + \tilde{1}^I x_2 - \tilde{1}^I x_3 \le \tilde{2}^I \\ -\tilde{1}^I x_1 - \tilde{1}^I x_2 - \tilde{1}^I x_3 \le -\tilde{1}^I & -\tilde{1}^I x_1 + \tilde{1}^I x_2 + \tilde{1}^I x_3 \le \tilde{1}^I \\ \tilde{1}^I x_1 - \tilde{1}^I x_2 + \tilde{1}^I x_3 \le \tilde{4}^I & x_1, x_2, x_3 \ge 0 \end{array} \right\} \tag{24}$$

Where, the triangular intuitionistic fuzzy number for all coefficients of objective and constrains functions in Eqs. (24) are $\tilde{7}^I = (6,7,8;5,7,9)$, $\tilde{3}^I = (1,3,5;0,3,6)$, $\tilde{4}^I = (2,4,6;0,4,8)$, $\tilde{1}^I = (0,1,2;-1,1,3)$, $\tilde{5}^I = (3,5,7;1,5,9)$, $\tilde{2}^I = (1,2,3;0,2,4)$.

Solution. Step 1: Based on the accuracy ranking method, the intuitionistic fuzzy multi-objective three-level programming (MO-TLP) problem Eqs. (24) is reduced into equivalent crisp MO-TLP problem as follows:

$$\max_{x_1}(z_{11}(\bar{x}) = 7x_1 + 3x_2 - 4x_3; z_{12}(\bar{x}) = x_1 + 3x_2 + 4x_3) \quad \text{[1st-}Level\text{]}$$

given x_1, where x_2 and x_3 solves

$$\max_{x_2}(z_{21}(\bar{x}) = 5x_1 - 2x_2 + x_3; z_{22}(\bar{x}) = x_1 + x_2 + x_3; z_{23}(\bar{x}) = -2x_1 + x_2 + 2x_3) \ \text{[2nd-}Level\text{]}$$

given x_1, x_2, where x_3 solve

$$\max_{x_3}(z_{31}(\bar{x}) = 3x_1 - 2x_2 + 2x_3; z_{32}(\bar{x}) = 5x_2 + 4x_3) \quad \text{[3rd-}Level\text{]}$$

Subject to:

$$S^c = \left\{ \begin{array}{lll} x_1 + x_2 + x_3 \le 5, & x_1 + x_2 - x_3 \le 2, & -x_1 - x_2 - x_3 \le -1; \\ -x_1 + x_2 + x_3 \le 1, & x_1 - x_2 + x_3 \le 4, & x_1, x_2, x_3 \ge 0. \end{array} \right\} \tag{25}$$

Step 2, 3, 4 & 5: Using the individual best solution of all objective functions at all levels, construct payoff table to get upper, lower tolerance limit and aspiration levels as shown in Table 5.2.

Step 6: Build the linear membership and non-membership of $z_{11}(\bar{x})$& $z_{12}(\bar{x})$ as: $\mu_{11}(z_{11}(\bar{x})) = \frac{7x_1+3x_2-4x_3-3.5}{11.5}$, $\nu_{11}(z_{11}(\bar{x})) = \frac{12.7-7x_1-3x_2+4x_3}{9.2}$, $\mu_{12}(z_{12}(\bar{x})) = \frac{x_1+3x_2+4x_3-7.5}{6.5}$, $\nu_{12}(z_{12}(\bar{x})) = \frac{13.6-x_1-3x_2-4x_3}{6.1}$

Step 7: Solve **Phase-I IFGP model for DM_1 MOP problem of Example 2** (5.2)**:** see the equation in Box IV.

Using LING-19 software solve the above model we obtained optimal solution $\bar{x}^*_1 = (x^*_{11}, x^*_{12}, x^*_{13}) = (2, 1.5, 1.5)$.

Step 8: $\mu_{11}(z_{11}(\bar{x}^*_1)) = 0.782, \mu_{11}(z_{12}(\bar{x}^*_1)) = 0.769 < 1$. Therefore, the obtained solution is both MN-Pareto optimal and Pareto optimal solution to DM_1 MOP problem.

Step 9, 10 & 11: First-level decision-maker set positive and negative tolerance for decision control variable $x^*_{11} = 2$ as $r^\mu_{11} = s^\mu_{11} = 1$ and $r^\nu_{11} = s^\nu_{11} = 1.5$. The weight of decision variable x_1 controlled by leader DM is $w^{-1\mu}_{11} = w^{-2\mu}_{11} = 1$ and $w^{+1\nu}_{11} = w^{+2\nu}_{11} = 0.667$.

Step 12: Since $t < 3$, go to step 5.

Step 5, 6 & 7: **Phase-I IFGP model for DM_2 MOP problem of Example 2** (5.2)**:** see the equation in Box V.

Solve the above single objective programming model using LINGO-19 software, we obtained an optimal solutions $\bar{x}^{**}_2 = (x^{**}_{21}, x^{**}_{22}, x^{**}_{23}) = (2, 1.5, 1.5)$.

Step 8: The membership value at this optimal solution of objective function for second-level decision-maker (DM_2) MOP problem are $\mu_{21}(z_{21}(\bar{x}^{**}_2)) = 1, \mu_{22}(z_{22}(\bar{x}^{**}_2)) = 0.8, \mu_{23}(z_{23}(\bar{x}^{**}_2)) = 1$. Therefore, this optimal solution is may not be Pareto optimal solution to DM_2 MOP problem, as we discussed on the previous section. To find Pareto optimal solution to DM_2 MOP problem we need to solve phase-II IFGP model as follows:

Phase-II IFGP model for DM_2 MOP problem of Example 2 5.2 see the equation in Box VI.

Solving the above phase-II model using LINGO-19 software and we get $\bar{x}^*_2 = (x^*_{21}, x^*_{22}, x^*_{23}) = (2, 0.5, 2.5)$. The optimal solution $\bar{x}^*_2$ dominate $\bar{x}^{**}_2$ because $z_{21}(\bar{x}^*_2) = 11.5 > z_{21}(\bar{x}^{**}_2) = 8.5, z_{22}(\bar{x}^*_2) = 5 = z_{22}(\bar{x}^{**}_2)$ and $z_{23}(\bar{x}^*_2) = 1.5 > z_{23}(\bar{x}^{**}_2) = 0.5$. Therefore, according to Theorem 4.2 and Jimenez and Bibao (2009) [30], $\bar{x}^*_2 = (2, 0.5, 2.5)$ satisfy both MN-Pareto and Pareto optimal solution to DM_2 MOP problem.

Step 9, 10 & 11: The second-level decision-maker decides positive and negative tolerance limit for decision control variable $x^*_{22} = 0.5$ as $r^\mu_2 = \frac{2}{3}, s^\mu_2 = 1.5$ and $r^\nu_2 = 1 = s^\nu_2$.

Step 12: Since $t = 3$, go to step 13.

Step 13: **Phase-I IFGP model for DM_3 MOP problem of Example 2** (5.2)**:** see the equation in Box VII.

Final step: Solving the above Phase-I IFGP model for DM_3 MOP problem using LINGO-19 software and we get $\bar{x}^* = (2, 1.5, 1.5)$ is Pareto optimal solution to numerical Example 2 (5.2) with the decision of DMs.

Table 5.2
Individual maximum values, minimum values, upper tolerances, lower tolerances, and goals for each objective functions.

	$z_{11}(\bar{x})$	$z_{12}(\bar{x})$	$z_{21}(\bar{x})$	$z_{22}(\bar{x})$	$z_{23}(\bar{x})$	$z_{31}(\bar{x})$	$z_{32}(\bar{x})$
$\max_{\bar{x}\in S^c} z_{tj}(\bar{x})$	17	14	16	5	2	11	13.5
$\min_{\bar{x}\in S^c} z_{tj}(\bar{x})$	−4	1	−2	1	−4	−2	0
Upper tolerance U^μ_{tj}	15	14	8.5	6	0.5	11	13
Upper tolerance U^ν_{tj}	12.7	13.6	7	5.4	0	10.2	12.8
Lower tolerance $L^\nu_{tj} = L^\mu_{tj}$	3.5	7.5	1	1	−4	6	8.5
Goals	15	14	8.5	6	0.5	11	13
W^μ_{tj}	0.087	0.154	0.133	0.2	0.2	0.182	0.2
W^ν_{tj}	0.108	0.164	0.167	0.25	0.227	0.208	0.212

5.2.1. Comparison with FGP method for Example 2 (5.2)

Recently, several researchers develop the FGP algorithm to solve different domain of MO-MLP problem, [1,6,8,16,18]. The solution of the numerical example 2 (5.2): results by using FGP algorithm is $\hat{x} = (2, 1, 2)$. A comparison shown in Table 5.2.1 between the solution $\bar{x}^* = (2, 1.5, 1.5)$ obtained using proposed two-phase IFGP method and $\hat{x} = (2, 1, 2)$ obtained using FGP algorithm. The relation between the distance measure function of $\bar{x}^* = (2, 1.5, 1.5)$ and $\hat{x} = (2, 1, 2)$ is $d(\bar{x}^*) = 0.735402 < d(\hat{x}) = 0.753394$. This clearly show that the optimal solution $\hat{x}$ is dominated by proposed Pareto-optimal solution $\bar{x}^*$.

6. Conclusion

A two-phase IFGP method is presented in this study for finding a Pareto optimal solution to a linear Multi-objective Multilevel Programming (MO-MLP) problem in an intuitionistic fuzzy environment. Intuitionistic fuzzy optimization approaches are one of the most effective tools for modeling optimization problems in an imprecise environment,

$$\min 0.087D_{11}^{-\mu}+0.108D_{11}^{+\nu}+0.154D_{12}^{-\mu}+0.164D_{12}^{+\nu}$$
$$\text{Subject to: }\left\{\begin{array}{ll} -7x_1-3x_2+4x_3-11.5D_{11}^{-\mu}\le -15, & -7x_1-3x_2+4x_3-9.2D_{11}^{+\nu}\le -12.7 \\ -x_1-3x_2-4x_3-6.5D_{12}^{-\mu}\le -14, & -x_1-3x_2-4x_3-6.1D_{12}^{+\nu}\le -13.6 \\ D_{11}^{-\mu},D_{11}^{+\nu},D_{12}^{-\mu},D_{12}^{+\nu}\ge 0, & \text{With the system constraint (25)} \end{array}\right\} \quad (26)$$

Box IV.

$$\min 0.087D_{11}^{-\mu}+0.108D_{11}^{+\nu}+0.154D_{12}^{-\mu}+0.164D_{12}^{+\nu}+0.133D_{21}^{-\mu}+0.167D_{21}^{+\nu}+0.2D_{22}^{-\mu}$$
$$+0.25D_{22}^{+\nu}+0.2D_{23}^{-\mu}+0.227D_{23}^{+\nu}+d_1^{-1\mu}+d_1^{-2\mu}+0.667d_1^{+1\nu}+0.667d_1^{+2\nu}$$
$$\text{Subject to: }\left\{\begin{array}{ll} -5x_1+2x_2-x_3-7.5D_{21}^{-\mu}\le -8.5, & -5x_1+2x_2-x_3-6D_{21}^{+\nu}\le -7, \\ -x_1-x_2-x_3-5D_{22}^{-\mu}\le -6, & -x_1-x_2-x_3-4.4D_{22}^{+\nu}\le -5.4, \\ 2x_1-x_2-2x_3-5D_{23}^{-\mu}\le -0.5, & 2x_1-x_2-2x_3-4D_{23}^{+\nu}\le 0, \\ -x_1-d_1^{-1\mu}\le -2, x_1-d_1^{-2\mu}\le 2, & -x_1-1.5d_1^{+1\nu}\le -2 \\ x_1-1.5d_1^{+2\nu}\le 2, & d_1^{-1\mu},d_1^{-2\mu},d_1^{+1\nu},d_1^{+2\nu}\ge 0, \\ D_{tj}^{-\mu},D_{tj}^{+\nu}\ge 0, t=1,2, j=1,2,3, & \text{With the system constraint (26)} \end{array}\right\} \quad (27)$$

Box V.

$$\max 0.016D_1+0.4D_2$$
$$\text{Subject to: }\left\{\begin{array}{ll} 5x_1-2x_2+x_3-D_1=8.5, x_1+x_2+x_3=5, & -2x_1+x_2+2x_3-D_2=0.5, \\ -7x_1-3x_2+4x_3-11.5D_{11}^{-\mu}\le -15, & -7x_1-3x_2+4x_3-9.2D_{11}^{+\nu}\le -12.7, \\ -x_1-3x_2-4x_3-6.5D_{12}^{-\mu}\le -14, & -x_1-3x_2-4x_3-6.1D_{12}^{+\nu}\le -13.6, \\ -x_1-d_1^{-1\mu}\le -2, x_1-d_1^{-2\mu}\le 2, & -x_1-1.5d_1^{+1\nu}\le -2, x_1-1.5d_1^{+2\nu}\le 2, \\ D_1,D_2,D_{11}^{-\mu},D_{11}^{+\nu}\ge 0, & \text{With the system constraint (25).} \end{array}\right\} \quad (28)$$

Box VI.

$$\min 0.087D_{11}^{-\mu}+0.108D_{11}^{+\nu}+0.154D_{12}^{-\mu}+0.164D_{12}^{+\nu}+0.133D_{21}^{-\mu}+0.167D_{21}^{+\nu}+0.2D_{22}^{-\mu}$$
$$+0.25D_{22}^{+\nu}+0.2D_{23}^{-\mu}+0.227D_{23}^{+\nu}+0.182D_{31}^{-\mu}+0.208D_{31}^{+\nu}+0.2D_{32}^{-\mu}+0.212D_{32}^{+\nu}$$
$$+d_1^{-1\mu}+d_1^{-2\mu}+0.667d_1^{+1\nu}+0.667d_1^{+2\nu}+1.5d_2^{-1\mu}+0.67d_2^{-2\mu}+d_2^{+1\nu}+d_2^{+2\nu} \quad (29)$$
$$\text{Subject to: }\left\{\begin{array}{ll} -3x_1+2x_2-2x_3-5.5D_{31}^{-\mu}\le -11, & -3x_1+2x_2-2x_3-4.8D_{31}^{+\nu}\le -10.2, \\ -5x_2-4x_3-4.5D_{32}^{-\mu}\le -13, & -5x_2-4x_3-3.6D_{32}^{+\nu}\le -12.8, \\ -2x_2-d_2^{-1\mu}\le -3, x_2-d_2^{-2\mu}\le 1.5, & -x_2-d_2^{+1\nu}\le -0.5, x_2-d_2^{+2\nu}\le 0.5 \\ D_{tj}^{-\mu},D_{tj}^{+\nu},d_2^{-1\mu},d_2^{-2\mu},d_2^{-1\nu},d_2^{-2\nu}\ge 0 & \text{With the system constraint (27)} \end{array}\right\}$$

Box VII.

Table 5.2.1
Comparison of Pareto-optimal solution for numerical example 2 (5.2) based on the suggested Two-phase IFGP method and the FGP approach in [1,6,8,16,18].
Pareto-optimal solution to numerical example.

$\bar{x}^*, \hat{x}$	$d(\bar{x})$	$z_{11}(\bar{x})$	$z_{12}(\bar{x})$	$z_{21}(\bar{x})$	$z_{22}(\bar{x})$	$z_{23}(\bar{x})$	$z_{31}(\bar{x})$	$z_{32}(\bar{x})$
$\max_{\bar{x}\in S^c} z_{tj}(\bar{x})$	0	17	14	16	5	2	11	13.5
Proposed method	0.735402	12.5	12.5	8.5	5	0.5	6	13.5
FGP approach	0.753394	9	13	10	5	1	8	13

and they produce more pleasing outcomes than crisp and fuzzy optimization strategies. In the proposed method, the crisp linear MO-MLP problem is obtained using the accuracy ranking method. And then we solve the upper-level decision-maker MOP problem using the proposed approach. Then the degree of optimality for decision variable control by the top-level is measured using membership and non-membership to relax the feasible region of the lower-level decision-maker MOP problem. Then, it continues until the last level decision-maker MOP problem is solved. In this procedure, the phase-II IFGP model is used to develop a compensatory solution that meets the MN-Pareto optimum solution as well as the Pareto optimal solution. Finally, two numerical examples were solved to demonstrate the proposed method for comparisons and the resulting optimal solution for the Two-phase IFGP method was better compared to Baky (2010) [1], Arbaiy & Watada (2012) [8] in the FGP approach.

In the future, the proposed method could be utilized to tackle nonlinear and quadratic MO-MLP issues in an intuitionistic fuzzy environment.

CRediT authorship contribution statement

Demmelash Mollalign: Conceptualization, Investigation, Formal analysis, Methodology, Visualization, Literature searches, Software, Data curation, Writing – original draft. **Allen Mushi:** Managed the literature searches, Data curation, Validation, Supervision, Writing – review & editing. **Berhanu Guta:** Managed the literature resources, Software, Validation, Supervision, Project administration.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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